3. [14 points] A farmer notices that a population of grasshoppers is growing at undesirable levels in his crop. He decides to hire the services of a pest control company. They offer the farmer a pesticide capable of eliminating the grasshoppers at a rate of 1 thousand grasshoppers per week. In the absence of pesticides, it is estimated that the grasshopper population grows at a rate of 20 percent every week. Let \( P(t) \) be the number of grasshoppers (in thousands) \( t \) weeks after the pesticide is applied to the crop. Then \( P(t) \) satisfies

\[
\frac{dP}{dt} = \frac{P}{5} - 1.
\]

Suppose there are \( P_0 \) thousand grasshoppers in the crop at the time the pesticide is applied in the crop.

**a.** [8 points] Find a formula for \( P(t) \) in terms of \( t \) and \( P_0 \).

**Solution:**

\[
\begin{align*}
\frac{dP}{dt} &= \frac{P}{5} - 1, \\
\frac{dP}{dt} &= \frac{1}{5}(P - 5) \\
\frac{dP}{P - 5} &= \frac{1}{5}dt \\
\ln|P - 5| &= \frac{1}{5}t + C \\
P - 5 &= Be^{\frac{1}{5}t} \\
P(0) = P_0 &= 5 + B \\
B &= P_0 - 5. \\
P(t) &= 5 + (P_0 - 5)e^{\frac{1}{5}t}.
\end{align*}
\]

**b.** [3 points] Does the differential equation have any equilibrium solutions? List each equilibrium solution and determine whether it is stable or unstable. **Justify your answer.**

**Solution:** Equilibrium solutions: \( P(t) = 5 \). The equilibrium is unstable since for \( P_0 > 5 \) \( P(t) \) increases and for \( P_0 < 5 \) \( P(t) \) decreases.

**c.** [3 points] Does the effectiveness of the pesticide depend on \( P_0 \)? That is, is the pesticide guaranteed to eliminate the grasshopper population regardless of the value of \( P_0 \), or are there some values of \( P_0 \) for which the grasshoppers will survive? If so, determine these values of \( P_0 \).

**Solution:** The pesticide is effective if \( P_0 < 5 \) and ineffective if \( P_0 \geq 5 \).