4. [12 points] Another farmer notices the plague of grasshoppers has spread to his crop. He also visits the pest control company and requests a cheaper pesticide. This new pesticide is capable of eliminating the grasshoppers at a rate that decreases with time. Specifically, the rate at which grasshoppers are killed is given by the function $f(t) = \frac{3}{10}(4 - t)$ in thousands of grasshoppers per week at $t$ weeks after the pesticide application. There is no pesticide remaining after 4 weeks. Suppose there are 3000 grasshoppers at the time the pesticide is applied.

Let $Q(t)$ the population of grasshoppers (in thousands) $t$ weeks after this cheaper pesticide is applied to the crop. Then for $0 \leq t \leq 4$, $Q(t)$ satisfies

$$\frac{dQ}{dt} = \frac{Q}{5} - f(t).$$

a. [1 point] Is this differential equation separable?

\textit{Solution:} No

b. [7 points] Using Euler’s method, fill the table with the amount of grasshoppers (in thousands) in the crop during the first week. Show all your computations.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>$\frac{1}{2}$</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q(t)$</td>
<td>3</td>
<td>2.7</td>
<td>2.445</td>
</tr>
</tbody>
</table>

\textit{Solution:}

$Q(0) = 3$ and $\Delta Q = \frac{1}{2}$, then

$Q_0 = 3$.

$Q_1 = Q_0 + (\frac{Q_0}{5} - f(0))\Delta Q = 2.7$

$Q_2 = Q_1 + (\frac{Q_1}{5} - f(\frac{1}{2}))\Delta Q = 2.445$
(problem 4 continued)

Use the slope field of the differential equation satisfied by $Q(t)$ to answer the following questions.

\[ \begin{align*}
\text{c. [2 points]} & \quad \text{Does this equation have any equilibrium solutions in the region shown? List each equilibrium solution and determine whether it is stable or unstable. Justify your answer.} \\
\text{\underline{Solution:}} & \quad \text{No equilibrium solutions. There is no } y \text{ value at which all the lines have slope 0.}
\end{align*} \]

\[ \begin{align*}
\text{d. [2 points]} & \quad \text{If the farmer’s goal is to kill all the grasshoppers in his crop, will the pesticide be effective in this case? Draw the solution } Q(t) \text{ on the slope field.} \\
\text{\underline{Solution:}} & \quad \text{No}
\end{align*} \]