4. [12 points] Another farmer notices the plague of grasshoppers has spread to his crop. He also visits the pest control company and requests a cheaper pesticide. This new pesticide is capable of eliminating the grasshoppers at a rate that decreases with time. Specifically, the rate at which grasshoppers are killed is given by the function $f(t) = \frac{3}{10}(4-t)$ in thousands of grasshoppers per week at t weeks after the pesticide application. There is no pesticide remaining after 4 weeks. Suppose there are 3000 grasshoppers at the time the pesticide is applied.

Let $Q(t)$ the population of grasshoppers (in thousands) t weeks after this cheaper pesticide is applied to the crop. Then for $0 \leq t \leq 4$, $Q(t)$ satisfies

$$\frac{dQ}{dt} = \frac{Q}{5} - f(t).$$

a. [1 point] Is this differential equation separable?

Solution: No

b. [7 points] Using Euler's method, fill the table with the amount of grasshoppers (in thousands) in the crop during the first week. Show all your computations.

t	0	$\frac{1}{2}$	1
$Q(t)$	3	2.7	2.445

Solution:

$Q(0) = 3$ and $\Delta Q = \frac{1}{2}$, then

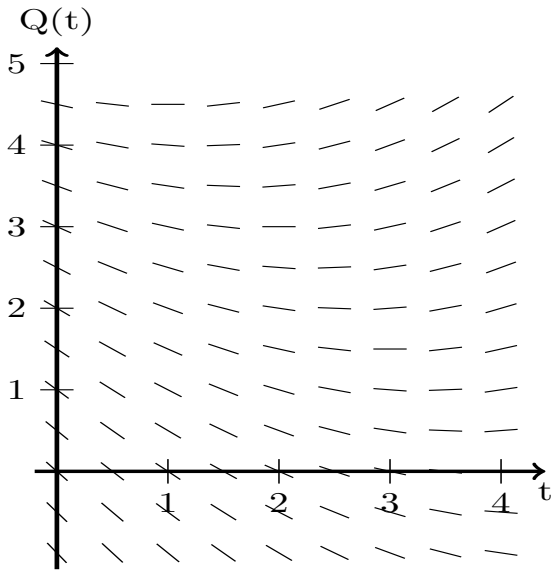
$Q_0 = 3.$

$Q_1 = Q_0 + (\frac{Q_0}{5} - f(0))\Delta Q = 2.7$

$Q_2 = Q_1 + (\frac{Q_1}{5} - f(\frac{1}{2}))\Delta Q = 2.445$

(problem 4 continued)

Use the slope field of the differential equation satisfied by $Q(t)$ to answer the following questions.



c. [2 points] Does this equation have any equilibrium solutions in the region shown? List each equilibrium solution and determine whether it is stable or unstable. **Justify your answer**.

Solution: No equilibrium solutions. There is no y value at which all the lines have slope 0.

d. [2 points] If the farmer's goal is to kill all the grasshoppers in his crop, will the pesticide be effective in this case? Draw the solution $Q(t)$ on the slope field.

Solution: No