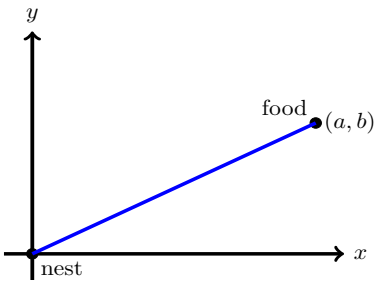
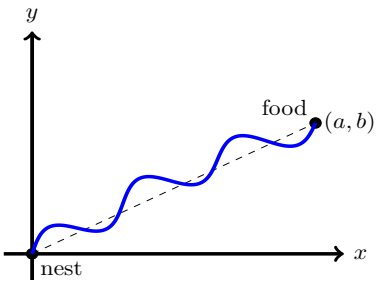
5. [10 points]
When an ant finds food, it leaves a trail of chemicals from its nest to the food source. Other ants follow this chemical trail using their two antennae. When an ant possesses both antennae, it will walk in a straight line to the food. If you remove (amputate) the left antenna of an ant, it will walk in a pattern like the one shown in the second figure.



Healthy



Amputated

a. [4 points] Write a parametric equation for the path of a healthy ant that starts at its nest at (0, 0) when $t = 0$ and arrives at the food at (a, b) when $t = 1$.

Solution: $x(t) = at$ and $y(t) = bt$.

b. [6 points] Suppose the parametric equation for the amputated ant is given by

$$x = x(t) \qquad y = y(t).$$

Assume the ant starts walking at $t = 0$, arrives at the food at $t = 1$, and never pauses or backtracks. For each blank below, determine whether the number on the left is greater than, less than, or equal to the number on the right, and fill the blank with the symbol $>$, $<$, or $=$ respectively. **Justify your answers**.

Solution:

$$\frac{y'(1)}{x'(1)} \quad > \quad 0$$

Answer 1: The slope of the tangent line to the curve at (a, b) is positive

Answer 2: The quotient is undefined since the ant stopped.

$$x'(c) \ \text{ (for any } 0 < c < 1) \quad > \quad 0$$

The ant is always moving to the right

$$\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt \quad > \quad \sqrt{a^2 + b^2}$$

The length of the line is shorter than the length of the curve