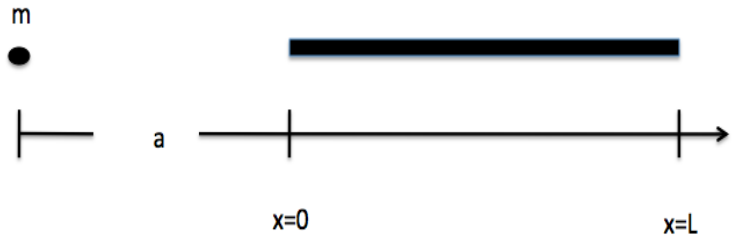
7. [7 points] A rod of length L meters has mass density δ_0, where $0 \leq x \leq L$ represents the position in meters along the rod measured from its left endpoint. The force of gravitational attraction F between the rod and a particle of mass m lying in the same line as the rod at a distance a is given by

$$F = \int_0^L \frac{Gm\delta_0}{(a+x)^2} dx.$$

where G is the constant of gravitation.



In certain cases (when the mass of the particle is small and the rod is long), you can assume that the rod has infinite length. Calculate the gravitational force between a rod of infinite length and a particle of mass m which is a meters away (arranged as shown above).

Solution:

$$\begin{aligned}
\int_0^\infty \frac{Gm\delta_0}{(a+x)^2} dx &= Gm\delta_0 \lim_{b\to\infty} \int_0^b \frac{1}{(a+x)^2} dx \\
&= Gm\delta_0 \lim_{b\to\infty} \frac{-1}{(a+x)}\Big|_0^b \\
&= Gm\delta_0 \lim_{b\to\infty} \frac{-1}{(a+b)} + \frac{1}{a} \\
&= \frac{Gm\delta_0}{a}
\end{aligned}$$