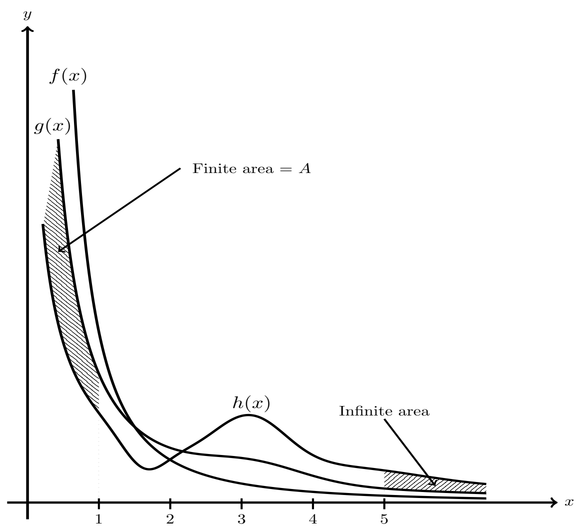
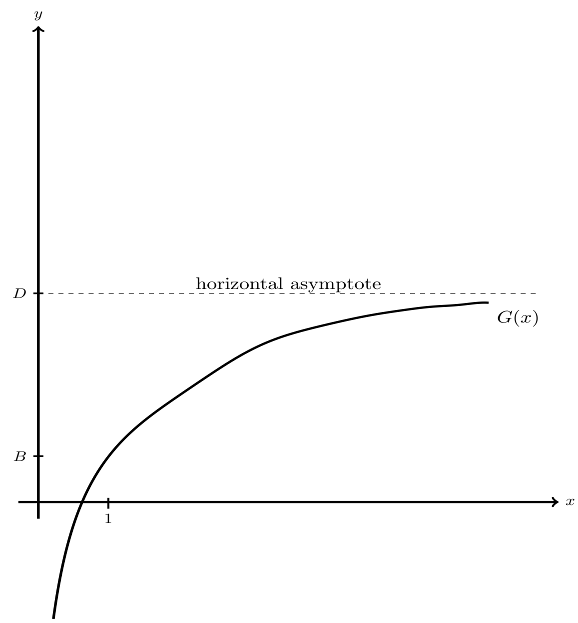
8. [15 points] Graphs of f, g and h are below. Each function is positive, is continuous on $(0, \infty)$, has a horizontal asymptote at $y = 0$ and has a vertical asymptote at $x = 0$. The area between $g(x)$ and $h(x)$ on the interval $(0, 1]$ is a finite number A, and the area between $g(x)$ and $h(x)$ on the interval $[5, \infty)$ is infinite. On the right is a graph of an antiderivative $G(x)$ of $g(x)$. It also has a vertical asymptote at $x = 0$.
Use the information in these graphs to determine whether the following three improper integrals **converge**, **diverge**, or whether there is **insufficient information to tell**. You may assume that f, g and h have no intersection points other than those shown in the graph. **Justify all your answers**.





a. [3 points] $\displaystyle\int_1^\infty h(x)dx$

Solution: Diverges

$$\int_1^\infty h(x)dx = \int_1^5 h(x)dx + \int_5^\infty h(x)dx = \text{finite integral} + \text{divergent integral}$$

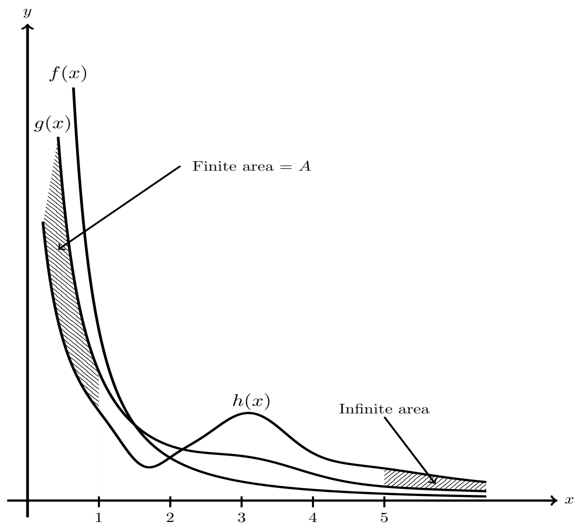
b. [4 points] $\displaystyle\int_0^1 g(x)dx$

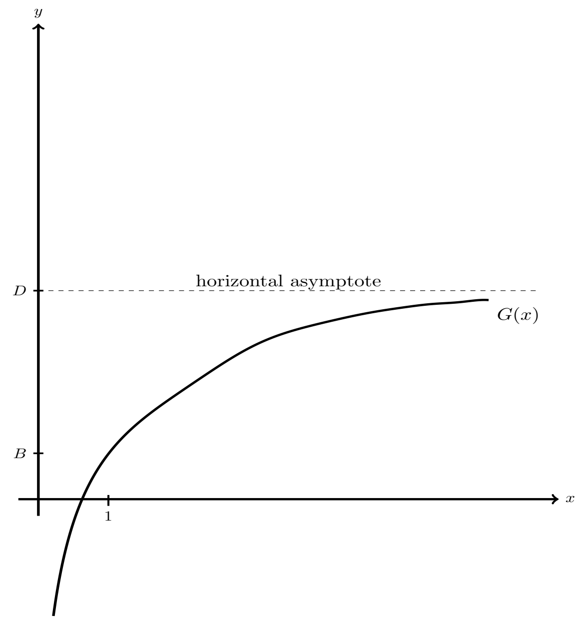
Solution: Diverges

$$\int_0^1 g(x)dx = \lim_{b\to 0^+} \int_b^1 g(x)dx = \lim_{b\to 0^+} G(x)|_b^1 = \lim_{b\to 0^+} G(1) - G(b) = \infty \text{ Diverges}$$

(problem 8 continued)

These graphs are the same as those found on the previous page.





c. [3 points] $\int_0^1 h(x)dx$

Solution: Diverges since

$$\int_0^1 h(x)dx = \int_0^1 g(x)dx - \int_0^1 (g(x) - h(x))dx = \text{ divergent integral+ finite integral}$$

d. [5 points] If $f(x) = 1/x^p$, what are all the possible values of p? **Justify your answer**.

Solution:

$$\int_1^\infty g(x)dx = \lim_{b\to\infty} \int_1^b g(x)dx$$
$$= \lim_{b\to\infty} G(b) - G(1) = D - B \text{ converges}$$
$$\int_3^\infty f(x)dx < \int_3^\infty g(x)dx \text{ convergent integral}$$

Hence $p > 1$.