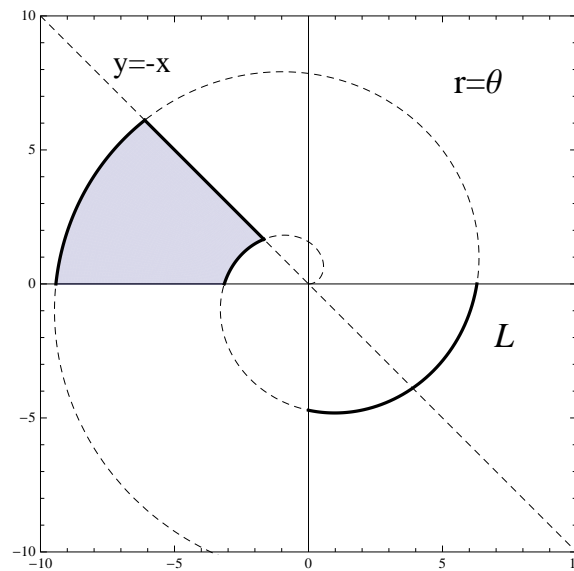


2. [11 points] Consider the graph of the spiral $r = \theta$ for $\theta \geq 0$.



In the following questions, write an expression involving definite integrals that computes the values of the following quantities (you do not need to evaluate any integrals) :

- a. [4 points] The length of the arc L .

Solution: The arc length of a polar curve is given by the formula

$$\int_{\theta_1}^{\theta_2} \sqrt{[r(\theta)]^2 + [r'(\theta)]^2} d\theta.$$

Therefore,

$$\text{Length of } L = \int_{3\pi/2}^{2\pi} \sqrt{\theta^2 + 1} d\theta.$$

- b. [7 points] The area of the shaded region.

Solution: The of the region inside of a polar curve is

$$\frac{1}{2} \int_{\theta_1}^{\theta_2} [r(\theta)]^2 d\theta.$$

We have to take the outer area minus the inner area. We can write the line $y = -x$ as $\theta = \frac{3\pi}{4} + 2\pi k$ for some k . The outer curve is on the second pass around the origin, and the inner curve is on the first time around the origin. Wo we get

$$A = \frac{1}{2} \int_{11\pi/4}^{3\pi} \theta^2 d\theta - \frac{1}{2} \int_{3\pi/4}^{\pi} \theta^2 d\theta.$$