

1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Consider the parametric equation given by $x = a(1 + t^2)$ and $y = 1 - t^3$, where $a > 0$. Then the curve is concave up at the point $(x, y) = (2a, 0)$.

True False

b. [2 points] Let $f(x)$ be a continuous function satisfying $\lim_{x \rightarrow \infty} f(x) = 0$. Then

$$\lim_{b \rightarrow \infty} \int_b^{\infty} f(x) dx = 0.$$

True False

c. [2 points] The point P whose polar coordinates $(r, \theta) = (1, \frac{\pi}{6})$ also has coordinates $(r, \theta) = (-1, \frac{7\pi}{6})$.

True False

d. [2 points] $\int_0^2 \ln(1 + t) dt$ is an improper integral.

True False

e. [2 points] All the solutions $y(t)$ of the differential equation $\frac{dy}{dt} = t^3$ are concave up.

True False

f. [2 points] The length of the parametric curve given by $x = \cos t$ and $y = \cos t + 1$ is $2\sqrt{2}$.

True False