1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Consider the parametric equation given by $x = a(1+t^2)$ and $y = 1-t^3$, where $a > 0$. Then the curve is concave up at the point $(x, y) = (2a, 0)$.

True **[False]**

Solution: For $t \neq 0$,

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{y'(t)}{x'(t)}\right)}{x'(t)} = \frac{\frac{d}{dt}\left(\frac{-3t^2}{2at}\right)}{2at} = \frac{\frac{d}{dt}\left(\frac{-3t^2}{2at}\right)}{2at} = \frac{\frac{-3}{2a}}{2at} = \frac{-3}{4a^2t}.$$

Since $(x(1), y(1)) = (2a, 0)$, then $\frac{d^2y}{dx^2}|_{t=1} = \frac{-3}{4a^2} < 0$ at that point. Hence the curve is concave down at $(2a, 0)$.

b. [2 points] Let $f(x)$ be a continuous function satisfying $\lim_{x\to\infty} f(x) = 0$. Then

$$\lim_{b\to\infty} \int_b^\infty f(x)dx = 0.$$

True **[False]**

Solution: If $\int_0^\infty f(x)dx$ diverges, then for any $b > 0$, $\int_b^\infty f(x)dx$ diverges. Then $\lim_{b\to\infty} \int_b^\infty f(x)dx \neq 0$.

c. [2 points] The point P whose polar coordinates $(r, \theta) = (1, \frac{\pi}{6})$ also has coordinates $(r, \theta) = (-1, \frac{7\pi}{6})$.

[True] False

Solution:

d. [2 points] $\int_0^2 \ln(1+t)\, dt$ is an improper integral.

True **[False]**

Solution: The function $\ln(1+t)$ is continuous on $[0, 2]$.

e. [2 points] All the solutions $y(t)$ of the differential equation $\frac{dy}{dt} = t^3$ are concave up.

[True] False

Solution: The solution $y(t)$ satisfies $\frac{d^2y}{dt^2} = 3t^2 \geq 0$ hence concave up.

f. [2 points] The length of the parametric curve given by $x = \cos t$ and $y = \cos t + 1$ is $2\sqrt{2}$.

[True] False

Solution: The length of the curve is L

$$L = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$
$$= \int_0^{\pi} \sqrt{(-\sin t)^2 + (-\sin t)^2} dt = \sqrt{2} \int_0^{\pi} \sin t \; dt = 2\sqrt{2}.$$