3. [10 points] The function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} + 2y = 2t \quad \text{with} \quad y(0) = 1.$$

a. [1 point] Can you use the method of separation of variables to solve this differential equation?  

Solution: No.

b. [4 points] Use Euler’s method with two steps to estimate $y(1)$. Fill the table with the values you find.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(t)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution: The equation can be rewritten as $\frac{dy}{dt} = 2t - 2y$

$y(0) = 1$ and $\Delta t = \frac{1}{2}$.

$y(\frac{1}{2}) \approx 1 + (2(0) - 2(1))\frac{1}{2} = 0$.

$y(1) \approx 0 + (2(0.5) - 2(0))\frac{1}{2} = \frac{1}{2}$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(t)$</td>
<td></td>
<td></td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

c. [5 points] For what values of $a$ and $b$ is the function $y(t) = ae^{-2t} + b + t$ a solution to the differential equation

$$\frac{dy}{dt} + 2y = 2t \quad \text{with} \quad y(0) = 1 \quad a = \text{__________,} \quad b = \text{__________}.$$

Solution: Using the initial condition $y(0) = 1$, you get $a + b = 1$.

Plugging into the differential equation

$$\frac{dy}{dt} + 2y = (-2ae^{-2t} + 1) + 2( ae^{-2t} + b + t) = 1 + 2b + 2t.$$

$\frac{dy}{dt} + 2y = 2t$ implies

$1 + 2b + 2t = 2t$.

$1 + 2b = 0$ then $b = -\frac{1}{2}$.

Hence $b = -\frac{1}{2}$ and $a = 1.5$. 