3. [10 points] The function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} + 2y = 2t \qquad \text{with} \qquad y(0) = 1.$$

a. [1 point] Can you use the method of separation of variables to solve this differential equation?

Solution: No.

b. [4 points] Use Euler's method with two steps to estimate $y(1)$. Fill the table with the values you find.

t	0		
$y(t)$			

Solution: The equation can be rewritten as $\frac{dy}{dt} = 2t - 2y$

$$y(0) = 1 \quad \text{and} \quad \Delta t = \frac{1}{2}.$$

$$y(\frac{1}{2}) \approx 1 + (2(0) - 2(1))\frac{1}{2} = 0.$$

$$y(1) \approx 0 + (2(0.5) - 2(0))\frac{1}{2} = \frac{1}{2}.$$

t	0	0.5	1
$y(t)$	1	0	$\frac{1}{2}$

c. [5 points] For what values of a and b is the function $y(t) = ae^{-2t} + b + t$ a solution to the differential equation

$$\frac{dy}{dt} + 2y = 2t \qquad \text{with} \qquad y(0) = 1$$

$a =$ ______________ $b =$ ______________.

Solution: Using the initial condition $y(0) = 1$, you get $a + b = 1$.
Plugging into the differential equation

$$\frac{dy}{dt} + 2y = (-2ae^{-2t} + 1) + 2(ae^{-2t} + b + t) = 1 + 2b + 2t.$$

$$\frac{dy}{dt} + 2y = 2t \text{ implies}$$

$$1 + 2b + 2t = 2t.$$

$$1 + 2b = 0 \text{ then } b = -\frac{1}{2}$$

.

Hence $b = -\frac{1}{2}$ and $a = 1.5$.