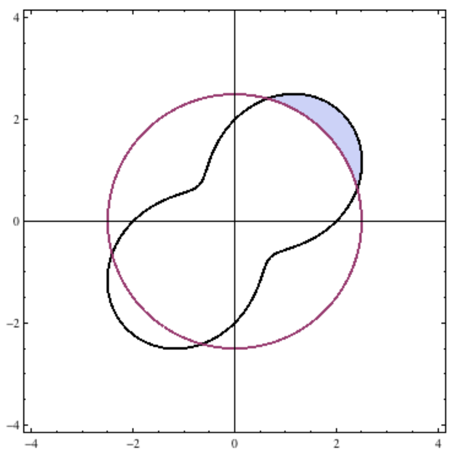
8. [14 points]

a. [6 points] Find a definite integral that computes the shaded area outside the circle $r = \frac{5}{2}$ and inside the curve given by $r = 2 + \sin 2\theta$ in the graph below.



Solution:

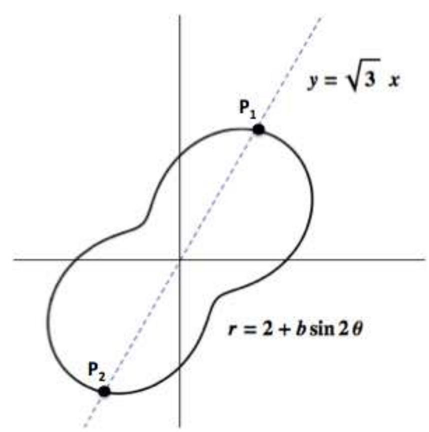
$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \left((2+\sin(2\theta))^2 - \left(\frac{5}{2}\right)^2 \right) d\theta.$$

Here we found the limits of integration by solving for where $5/2 = 2 + \sin 2\theta$ for θ in the first quadrant.

b. [4 points] Find the polar coordinates (r, θ) of the points where the line $y = \sqrt{3}\, x$ intersects the graph of $r = 2 + b \sin 2\theta$. Here the constant $0 < b < 2$. Your answers may include b.

P_1=__________________

P_2=__________________



Solution: We want $\tan\theta = \sqrt{3}$, so $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$. In the first case, we have

$$P_1 = (r,\theta) = \left(2 + b\sin\left(\frac{2\pi}{3}\right), \frac{\pi}{3}\right) = \left(2 + \frac{\sqrt{3}}{2}b, \frac{\pi}{3}\right).$$

$$P_2 = (r,\theta) = \left(2 + b\sin\left(\frac{8\pi}{3}\right), \frac{4\pi}{3}\right) = \left(2 + \frac{\sqrt{3}}{2}b, \frac{4\pi}{3}\right).$$

c. [4 points]

i) Find the equation in polar coordinates of the line $x = 0$.

Solution: $\theta = \dfrac{\pi}{2}$.

ii) Find the equation in polar coordinates of the line $y = 4$.

Solution: $y = r\sin\theta = 4$, so $r = \dfrac{4}{\sin\theta}$.