11. [10 points]

a. [5 points] Compute the improper integral $\displaystyle\int_0^1 \ln(x)\,dx$. Show your work.

Solution:
$\int_0^1 \ln(x)\,dx = \lim_{a\to 0^+} \int_a^1 \ln(x)\,dx = \lim_{a\to 0^+} x\ln(x)|_a^1 - \int_a^1 1\,dx = \lim_{a\to 0^+} -a\ln(a) - 1 + a$. Using either L'hopital's rule or the fact that polynomials dominate logarithms we have $\lim_{a\to 0^+} a\ln(a) = 0$. Therefore the integral is equal to -1.

b. [5 points] Use comparison of improper integrals to determine if the improper integral $\displaystyle\int_1^\infty \frac{\sin(x)+3}{x^2+2}$ converges or diverges. Show your work.

Solution:
We have the inequalities $\sin(x)+3 \le 4$ and $\frac{1}{x^2+2} \le \frac{1}{x^2}$. Therefore $\int_1^\infty \frac{\sin(x)+3}{x^2+1}\,dx \le \int_1^\infty \frac{4}{x^2}\,dx = 4\int_1^\infty \frac{1}{x^2}\,dx$. This integral is a p-integral with $p = 2 > 1$ so it converges. Therefore $\int_1^\infty \frac{\sin(x)+3}{x^2+2}\,dx$ converges by comparison.