7. [7 points] Bill has just built a brand new 90,000 L swimming pool. Bill is allergic to chlorine so instead he is using a filtration system to prevent algae from building up in the pool. Algae grows in the pool at a constant rate of 600 kg/day. The filtration system receives a constant supply of 70,000 L/day of water and returns the water to the pool with 6/7ths of the algae removed. Let $A(t)$ be the amount of algae in the pool in kilograms t days after Bill has filled the pool with fresh (algae free) water.

a. [5 points] Write down the differential equation satisfied by $A(t)$. Include the initial condition.

Solution:
$\frac{dA}{dt}$ = Rate in-Rate out. Rate in = 600 kg/day. Rate out=flow rate$\times$ concentration$\times$ fraction removed= $70,000 \times \frac{A}{90,000} \times 6/7 = \frac{2}{3}A$. Thus $\frac{dA}{dt} = 600 - \frac{2}{3}A$.

$$\frac{dA}{dt} = 600 - \frac{2}{3}A$$

Initial condition: $A(0) = 0$

b. [2 points] Find all the equilibrium solutions of the differential equation.

Solution:
We want to solve $\frac{dA}{dt} = 600 - \frac{2}{3}A = 0$. Therefore $A = 900$ is the only equilibrium solution.

8. [4 points] Consider the differential equation $y' = e^y$. Solve the differential equation with initial condition $y(0) = 1$.

Solution:
The equation $\frac{dy}{dx} = e^y$ is separable so we have $e^{-y}dy = dx$. Integrating both sides we get $-e^{-y} = x + c$. Solving the equation we get $y = -\ln(c - x)$. To solve for c we take $y(0) = -\ln(c) = 1$. Therefore $c = \frac{1}{e}$. So the solution is $y = -\ln(\frac{1}{e} - x)$.