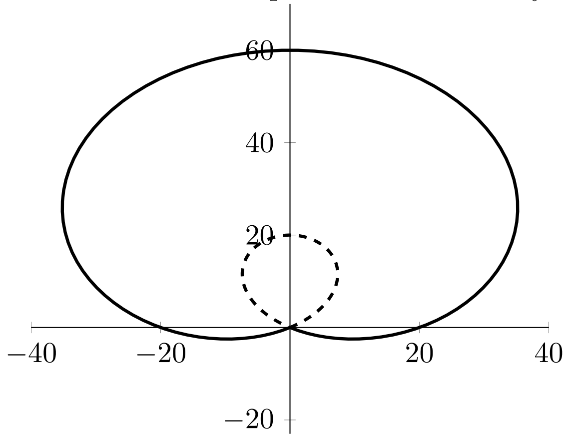
9. [10 points] Linda is designing a pond with a flat rock at one end. The rock plus the pond are in the shape of a cardioid. Plans for her pond design are depicted below. The cardioid has equation $r = 20 + 40\sin\theta$ where r is in feet and θ is in radians. The inner loop of the cardioid forms the shape of the rock and the outer loop forms the boundary of the pond.



a. [2 points] Find all values of θ between 0 and 2π for which $r = 0$.

Solution: First we set $r = 20 + 40\sin\theta = 0$. Therefore rearranging $\sin\theta = -\frac{1}{2}$. The solutions between 0 and 2π are $\theta = 7\pi/6, 11\pi/6$.

b. [4 points] Write an integral or sum of integrals which give(s) the perimeter of the boundary of the pond. Note this is the perimeter of the part of the cardioid drawn with a solid line.

Solution:
We will need to use the polar arc length formula so we need to calculate $r' = 40\cos\theta$. The arc length can be written as a single integral $\int_{-\pi/6}^{7\pi/6} \sqrt{(40\cos\theta)^2 + (20 + 40\sin\theta)^2}\, d\theta$. Writing the arc length as two integrals we get $\int_0^{7\pi/6} \sqrt{(40\cos\theta)^2 + (20 + 40\sin\theta)^2}\, d\theta + \int_{11\pi/6}^{2\pi} \sqrt{(40\cos\theta)^2 + (20 + 40\sin\theta)^2}\, d\theta$

c. [4 points] Write an integral or sum of integrals which give(s) the area of the top of the rock. Note this is the area enclosed by the dashed part of the cardioid.

Solution:
Now we need to use the polar area formula $\int_{7\pi/6}^{11\pi/6} \frac{1}{2}(20 + 40\sin\theta)^2\, d\theta$.