7. [9 points] A certain cosmological model predicts the evaporation rate of a black hole to be inversely proportional to its mass squared. This gives a first order differential equation

$$\frac{dM}{dt} = \alpha \frac{1}{M^2}$$

where $M = M(t)$ is the mass of the black hole in kg, t is time in seconds, and α is the constant of proportionality.

   a. [5 points] Find the general solution using separation of variables.

   b. [4 points] How long will it take for a black hole with initial mass 8×10^{22} kg, which is approximately the mass of the moon, to evaporate if $\alpha = -\frac{8}{3} \times 10^{17}$ kg^3/sec?