1. [16 points] Carla and Bobby run a race after spinning in circles for a good amount of time to make themselves dizzy. They start at the origin in the xy-plane and they race to the line $y = 5$. Assume the units of x and y are meters.
Bobby's position in the xy-plane t seconds after the races starts is

$$(-\sqrt{3}t\cos t, \frac{1}{\sqrt{3}}t\sin t)$$

and Carla's position in the xy-plane t seconds after the race starts is

$$(t\sin t, -t\cos t).$$

a. [4 points] Write an integral that gives the distance that Carla travels during the first two seconds of the race. Do not evaluate your integral.

Solution: We have

$$\frac{dx}{dt} = \sin(t) + t\cos(t),$$
$$\frac{dy}{dt} = -\cos(t) + t\sin(t).$$

The distance traveled by Carla in the first two seconds of the race is then given by $\int_0^2 \sqrt{(\sin(t) + t\cos(t))^2 + (t\sin(t) - \cos(t))^2}dt$.

b. [3 points] Find Carla's speed at $t = \pi$.

Solution: We have that Carla's speed is given by the function $\sqrt{(\sin(t) + t\cos(t))^2 + (t\sin(t) - \cos(t))^2}$, and so we need only plug in $t = \pi$ which gives us the value below.

Carla's speed at $t = \pi$ is $\sqrt{\pi^2 + 1}$ **m/sec**

c. [4 points] Carla and Bobby are so dizzy that they run into each other at least once during the race. Find the first time $t > 0$ that they run into each other, and give the point (x, y) where the collision occurs.

Solution: Setting the x and y coordinate functions equal gives us that collisions will occur when $\tan(t) = -\sqrt{3}$. The first time for $t > 0$ when this occurs is $2\pi/3$. Plugging this t value into either the equations for Bobby's or Carla's position will give the (x, y) coordinates given below for where the collision occurs.

They first run into each other at $t =$ $\frac{2\pi}{3}$

The collision occurs at $(x, y) =$ $(\frac{\sqrt{3}}{3}\pi, \frac{\pi}{3})$

d. [5 points] Bobby's phone flies out of his pocket at $t = \pi/2$. It travels in a straight line in the same direction as he was moving at $t = \pi/2$. Find the equation of this line in Cartesian coordinates.

Solution: Plug in $t = \dfrac{\pi}{2}$ to the parametric equations for Bobby's position to get that Bobby is at the point $P = \left(0, \dfrac{\pi}{2\sqrt{3}}\right)$ at $t = \dfrac{\pi}{2}$. We can find the slope of the curve at that point

$$\left.\frac{dy}{dx}\right|_P = \frac{\frac{dy}{dt}|_{t=\pi}}{\frac{dx}{dt}|_{t=\pi}} = \frac{\frac{1}{\sqrt{3}}\sin(\frac{\pi}{2}) + \frac{\pi}{2\sqrt{3}}\cos(\frac{\pi}{2})}{-\sqrt{3}\cos(\frac{\pi}{2}) + \frac{\sqrt{3}\pi}{2}\sin(\frac{\pi}{2})} = \frac{2}{3\pi}.$$

Since the line that gives the path of the phone is the same as the tangent line to the curve giving Bobby's motion at the point P, the equation of the line we want is that given below.

The equation for the line is $y = \dfrac{2}{3\pi}x + \dfrac{\pi}{2\sqrt{3}}$