2. [6 points] Your friend the goliath frog is going to decorate the boundary of his lily pad with a string of tiny flowers. The boundary of the lily pad is given by a portion of the curve \( r = 13 + 26 \cos(\theta) \) where \( r \) is measured in inches and \( \theta \) is measured in radians. The part of the curve that traces out the lily pad is shown below in the \( xy \)-plane.

\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} \sqrt{(13 + 26 \cos(\theta))^2 + (26 \sin(\theta))^2} \, d\theta + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{(13 + 26 \cos(\theta))^2 + (26 \sin(\theta))^2} \, d\theta.
\]

If the goliath frog is going to decorate only the part of the boundary of the lily pad for which \( x \leq 0 \), write an expression involving integrals for the length of the string of flowers required. Do not evaluate your integral.

Solution: The length of the string of flowers is given by

\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} \sqrt{(13 + 26 \cos(\theta))^2 + (26 \sin(\theta))^2} \, d\theta + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{(13 + 26 \cos(\theta))^2 + (26 \sin(\theta))^2} \, d\theta.
\]

3. [4 points] We can approximate the value of \( \ln(1.5) \) by using the fact that \( y = \ln(x) \) solves the differential equation

\[
\frac{dy}{dx} = \frac{1}{x}
\]

Approximate \( \ln(1.5) \) by using Euler’s method for the differential equation above with initial condition \( y(1) = 0 \) and with \( \Delta x = 0.25 \). Fill in the table with the \( y \)-values obtained at each step.

Solution: We are given that \( y(1) = 0 \). Using Euler’s method with \( \Delta x = 0.25 \) we compute

\[
y(1.25) \approx y(1) + y'(1)\Delta x = 0 + (1)(0.25) = 0.25, \\
y(1.50) \approx y(1.25) + y'(1.25)\Delta x \approx 0.25 + (0.8)(0.25) = 0.45.
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>1.25</td>
<td>0.25</td>
</tr>
<tr>
<td>1.50</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Thus, \( \ln(1.5) \approx 0.45 \).