4. [7 points] You have an object attached to the end of a spring and you are trying to study its motion. Using Newton's second law and Hooke's law your physics teacher determines the displacement x from equilibrium of the object is a solution to the differential equation

$$\frac{d^2x}{dt^2} + 2x = 0.$$

For what values of A, B, and ω is

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

a solution to the equation above satisfying the initial conditions $x(0) = 1$ and $x'(0) = 2$? Write your answers in the blanks provided and be sure to show all work.

Solution: We can compute that

$$x'(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

and

$$x''(t) = -A\omega^2\cos(\omega t) - B\omega^2\sin(\omega t) = -\omega^2 x(t).$$

Note that the second expression implies that x will solve the differential equation above if $\omega^2 = 2$. Therefore, we have $\omega = \sqrt{2}$.

The expression for x in the statement of the problem and the initial condition $x(0) = 1$ imply that $A = 1$. Our expression for x' above and the initial condition $x'(0) = 2$ imply that $B = 2/\omega = 2/\sqrt{2}$.

$A =$ 1

$B =$ $2/\sqrt{2}$

$\omega =$ $\sqrt{2}$