**9**. [10 points] Determine if the following integrals converge or diverge. If the integral converges, circle the word "converges" and give the **exact value** (i.e. no decimal approximations). If the integral diverges, circle "diverges". In either case, **you must show all your work and indicate any theorems you use**. Any direct evaluation of integrals must be done **without using a calculator.**

**a**. [5 points] $\displaystyle\int_0^1 \ln(x)\, dx$

$\boxed{\text{CONVERGES}}$ DIVERGES

*Solution:* We have

$$\begin{aligned}\int_0^1 \ln(x)dx &= \lim_{a\to 0+}\int_a^1 \ln(x)dx\\ &= \lim_{a\to 0+} x\ln(x) - x|_a^1\\ &= \lim_{a\to 0+} -1 - a\ln(a) + a.\end{aligned}$$

L'Hospital's Rule tells us that $\displaystyle\lim_{a\to 0+} a\ln(a) = 0$. Thus, $\displaystyle\int_0^1 \ln(x)dx = -1$

**b**. [5 points] $\displaystyle\int_2^\infty \frac{x+\sin x}{x^2-x}\, dx$

CONVERGES $\boxed{\text{DIVERGES}}$

*Solution:* We have

$$\begin{aligned}\sin(x) &\geq -1,\\ x+\sin(x) &\geq x-1,\\ \frac{x+\sin(x)}{x^2-x} &\geq \frac{x-1}{x^2-x} = \frac{1}{x}.\end{aligned}$$

We also know that $\displaystyle\int_2^\infty \frac{1}{x}dx$ diverges, as it is an improper integral of the form $\displaystyle\int_a^\infty \frac{1}{x^p}dx$ for $p \leq 1$. Thus, by the comparison test we have that $\displaystyle\int_2^\infty \frac{x+\sin x}{x^2-x}dx$ diverges.