2. [13 points] Leia and Han are imprisoned in a cell whose door is made out of steel and has a thickness of 3 feet. Luke uses his lightsaber to cut through the door in the shape of the curve given by the polar coordinates equation

\[ r = \frac{5}{3 + 2 \cos \left( \theta + \frac{\pi}{4} \right)} \]

where \( r \) is measured in feet.

a. [6 points] Write an expression involving integrals for the volume of the piece that Luke cuts out of the door.

\[ \text{Solution:} \quad 3 \cdot \int_{0}^{2\pi} \frac{1}{2} \left( \frac{5}{3 + 2 \cos \left( \theta + \frac{\pi}{4} \right)} \right)^2 d\theta \quad \text{ft}^3 \]
b. [7 points] Still considering the polar curve
\[ r = \frac{5}{3 + 2 \cos \left( \theta + \frac{\pi}{4} \right)} \]
graphed in the \( xy \)-plane, write an explicit expression involving integrals for the length of the part of the curve that lies \textbf{to the right} of the \( y \)-axis.

\textbf{Solution:}
\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left( \frac{5}{3 + 2 \cos \left( \theta + \frac{\pi}{4} \right)} \right)^2 + \left( \frac{10 \sin \left( \theta + \frac{\pi}{4} \right)}{3 + 2 \cos \left( \theta + \frac{\pi}{4} \right)} \right)^2} \, d\theta \text{ ft}
\]

Alternatively:
\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left( \frac{dx}{d\theta} \right)^2 + \left( \frac{dy}{d\theta} \right)^2} \, d\theta \text{ ft}
\]

where
\[ \frac{dx}{d\theta} = \frac{(-5 \sin \theta)(3 + 2 \cos(\theta + \pi/4)) + (5 \cos \theta)2 \sin(\theta + \pi/4)}{[3 + 2 \cos(\theta + \pi/4)]^2} \]
and
\[ \frac{dy}{d\theta} = \frac{(5 \cos \theta)(3 + 2 \cos(\theta + \pi/4)) + (5 \sin \theta)2 \sin(\theta + \pi/4)}{[3 + 2 \cos(\theta + \pi/4)]^2} \]