2. [13 points] Leia and Han are imprisoned in a cell whose door is made out of steel and has a thickness of 3 feet. Luke uses his lightsaber to cut through the door in the shape of the curve given by the polar coordinates equation

$$r = \frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}$$

where r is measured in feet.

a. [6 points] Write an expression involving integrals for the volume of the piece that Luke cuts out of the door.

Solution:

$$3 \cdot \int_0^{2\pi} \frac{1}{2}\left(\frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}\right)^2 d\theta \quad \text{ft}^3$$

b. [7 points] Still considering the polar curve

$$r = \frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}$$

graphed in the xy-plane, write an explicit expression involving integrals for the length of the **part** of the curve that lies **to the right** of the y-axis.

Solution:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left(\frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}\right)^2 + \left(\frac{10\sin\left(\theta + \frac{\pi}{4}\right)}{\left(3 + 2\cos\left(\theta + \frac{\pi}{4}\right)\right)^2}\right)^2}\, d\theta \quad \text{ft}$$

Alternatively:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta \quad \text{ft}$$

where

$$\frac{dx}{d\theta} = \frac{(-5\sin\theta)(3 + 2\cos(\theta + \pi/4)) + (5\cos\theta)2\sin(\theta + \pi/4)}{[3 + 2\cos(\theta + \pi/4)]^2}$$

and

$$\frac{dy}{d\theta} = \frac{(5\cos\theta)(3 + 2\cos(\theta + \pi/4)) + (5\sin\theta)2\sin(\theta + \pi/4)}{[3 + 2\cos(\theta + \pi/4)]^2}$$

