

3. [13 points] O-guk's playful son, O-ghan, is running on the xy -plane. His position t seconds after he begins running is

$$x = \sqrt{t} - 1 \qquad y = \sin(t) + 1.$$

Assume x and y are in meters.

- a. [3 points] Does O-ghan pass through the origin? Briefly justify.

Solution: $x = 0$ when $\sqrt{t} - 1 = 0$ so when $t = 1$. For this value of t , $y = \sin(1) + 1 \neq 0$. So he didn't pass through the origin.

- b. [4 points] How fast is O-ghan running at $t = 5$? Give your answer in **exact** form (i.e. no decimal approximations). Include **units**.

Solution:

$$\sqrt{\left(\frac{1}{2\sqrt{5}}\right)^2 + (\cos(5))^2} \quad \frac{m}{s}$$

- c. [6 points] Find an equation, in xy -coordinates, of the tangent line to his path at $t = 1$.

Solution: The slope of the tangent line is given by

$$m = \frac{dy/dt}{dx/dt} = \frac{\cos(1)}{\frac{1}{2\sqrt{1}}} = 2 \cos(1)$$

The equation of the tangent line is $y - (\sin(1) + 1) = 2 \cos(1)(x - 0)$ or equivalently

$$y = 2 \cos(1)x + \sin(1) + 1$$