

8. [13 points] Brienne is hiking, and the temperature of the air in  $^{\circ}\text{C}$  after she's traveled  $x$  km is a solution to the differential equation

$$y' + y \sin x = 0$$

- a. [7 points] Find the general solution of the differential equation.

*Solution:* Writing the equation as  $\frac{dy}{dx} = -y \sin x$  and separating the variables we get

$$\int \frac{1}{y} dy = \int -\sin x dx$$

$$\ln |y| = \cos x + C$$

$$y = Ae^{\cos x}$$

- b. [2 points] If the temperature was  $10^{\circ}\text{C}$  at the beginning of the hike, find  $T(x)$ , the temperature of the air in  $^{\circ}\text{C}$  after she's traveled  $x$  km. Show your work.

*Solution:* From (a),  $T(x) = Ae^{\cos x}$ . Since  $T(0) = 10$  we get  $A = \frac{10}{e}$ . Thus,

$$T(x) = \frac{10}{e} e^{\cos x}$$

- c. [4 points] Brienne traveled 7 km on the hike. Using the information given in (b), find the coldest air temperature she encountered on the hike. Give an **exact** answer (i.e. no decimal approximations).

*Solution:* We want to find the minimum of the function  $T(x) = \frac{10}{e} e^{\cos x}$  over the interval  $[0, 7]$ . The critical points are  $0, \pi, 2\pi$ . Checking the outputs of  $T$  at those points and the endpoint 7 we find that the minimum is  $T(\pi) = \frac{10}{e^2}$ .