1. [8 points] For each of the following, remember to show your work carefully.

a. [4 points] For which value(s) of k is the function

$$L(t) = e^{kt}$$

a solution to the differential equation $L'' - L' = 6L$?
If there are no such values, write "NONE" in the answer blank.

Solution: If $L(t) = e^{kt}$, then $L'(t) = ke^{kt}$ and therefore $L''(t) = k^2e^{kt}$. So if $L(t)$ is a solution to the given differential equation, we find that

$$k^2e^{kt} - ke^{kt} = 6e^{kt}.$$

Dividing by e^{kt} (which is never zero) gives

$$0 = k^2 - k - 6 = (k-3)(k+2).$$

So $L(t)$ is a solution to the differential equation when $k = -2$ and when $k = 3$.

Answer: $k =$ $-2, 3$

b. [4 points] For which value(s) of C is the function

$$y = (e^x - x - C)^{1/2}$$

a solution to the differential equation $2y\dfrac{dy}{dx} = y^2 + x$?
If there are no such values, write "NONE" in the answer blank.

Solution: Note that if $y = (e^x - x - C)^{1/2}$, then

$$\frac{dy}{dx} = \frac{1}{2}(e^x - x - C)^{-1/2}(e^x - 1)$$

and thus

$$2y\frac{dy}{dx} = (e^x - 1).$$

On the other hand,

$$y^2 + x = (e^x - x - C) + x = (e^x - C)$$

So if $y = (e^x - x - C)^{1/2}$ is a solution to the given differential eqution, then $e^x - 1 = e^x - C$ and thus $C = 1$.

Answer: $C =$ 1