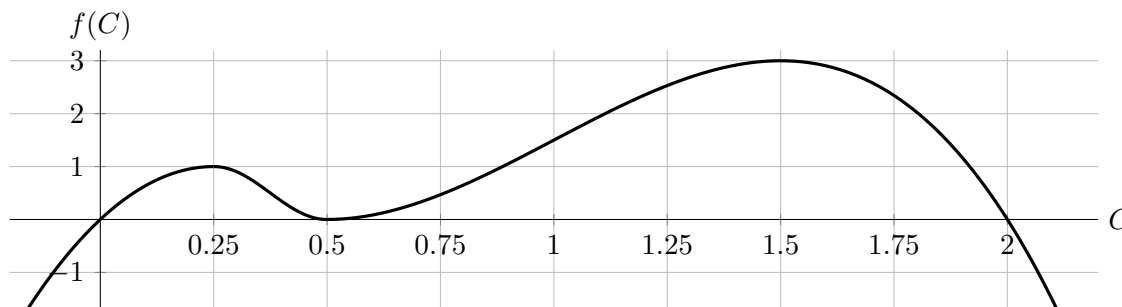


3. [5 points] Sasha and her friends are sipping lemonade on her boat when the boat begins to leak through a new hole in the bottom. Water begins to enter the boat at a constant rate of 1.5 gallons per minute. Immediately, they spring into action and begin to scoop the water out of the boat using lemonade pitchers that hold 0.25 gallons of water. That rate that the water is scooped, water, in scoops per minute, is proportional to the cube root of the volume of water currently in the boat, with constant of proportionality k . Let $W = W(t)$ be the volume of water in the boat, in gallons, t minutes after the leak begins. Write a differential equation that models $W(t)$, and give an appropriate initial condition.

Answer: Differential Equation: $\frac{dW}{dt} = 1.5 - 0.25kW^{1/3}$

Initial Condition: $W(0) = 0$

4. [6 points] Consider the differential equation $\frac{dC}{dt} = f(C)$ where $f(C)$ is the function graphed below.



- a. [4 points] Identify all equilibrium solutions of this differential equation. Then indicate which of these equilibrium solutions are stable. Write your answers on the answer blanks provided.

Answer: All Equilibrium Solutions: $C = 0, C = 0.5, C = 2$

Stable Equilibrium Solutions: $C = 2$

- b. [2 points] Suppose that a solution to this differential equation passes through a point with $C = 0.17$. For this solution, what will happen to the value of C as $t \rightarrow \infty$?

Solution: Note that $\frac{dC}{dt} > 0$ for $0.17 \leq C < 0.5$, and that $C = 0.5$ is an equilibrium solution. So C will increase from 0.17 and approach 0.5. That is, $\lim_{t \rightarrow \infty} C = 0.5$.