5. [10 points] Let $f(x)$ and $g(x)$ be two functions that are differentiable on $(0, \infty)$ with continuous derivatives and which satisfy the following inequalities for all $x \geq 1$:

$$\frac{1}{x} \leq f(x) \leq \frac{1}{x^{1/2}} \qquad \text{and} \qquad \frac{1}{x^2} \leq g(x) \leq \frac{1}{x^{3/4}}.$$

For each of the following, determine whether the integral always, sometimes, or never converges. Indicate your answer by circling the one word that correctly fills the answer blank. No justification is necessary. No credit will be awarded for unclear markings.

a. [2 points] $\displaystyle\int_1^\infty \sqrt{f(x)}\, dx$ _____ converges.

Always Sometimes **Never**

b. [2 points] $\displaystyle\int_3^\infty 4000 g(x)\, dx$ _____ converges.

Always **Sometimes** Never

c. [2 points] $\displaystyle\int_1^\infty f(x)g(x)\, dx$ _____ converges.

Always Sometimes Never

d. [2 points] $\displaystyle\int_5^\infty g'(x)e^{g(x)}\, dx$ _____ converges.

Always Sometimes Never

e. [2 points] $\displaystyle\int_1^\infty f'(x)\ln(f(x))\, dx$ _____ converges.

Always Sometimes Never