6. [13 points] Anderson and Glen decide to take a road trip starting from Venice Beach. They have no worries about getting anywhere quickly, as they enjoy each other’s company, so they take a very inefficient route. Suppose that Venice Beach is located at (0, 0) and that Anderson and Glen’s position \((x, y)\) (measured in miles) \(t\) hours after leaving Venice Beach is given by a pair of parametric equations \(x = f(t), y = g(t)\). A graph of \(f(t)\) and a formula for \(g(t)\) are given below. Note that \(f(t)\) is linear on the intervals \([0, 0.5], [0.5, 1.5],\) and \([2.5, 3]\).

\[
\begin{align*}
f(t) & = -3t^2 + 10t - 3 \\
g(t) & = -t^3 + 5t^2 - 3t
\end{align*}
\]

**Note:** For each of the following, your final answer should **not** involve the letters \(f\) and \(g\).

- **a.** [2 points] If their roadtrip last 3 hours, what are the \(x\)- and \(y\)-coordinates of their final destination?

  **Solution:** Note that at time \(t = 3\), we have \(x = f(3) = -2\) and \(y = g(3) = 9\). So the coordinates of their final destination are \((-2, 9)\).

- **b.** [3 points] At what speed are they traveling 2 hours into their trip?

  **Solution:** We have \(\frac{dx}{dt}_{t=2} = f'(2) = 0\) and \(\frac{dy}{dt}_{t=2} = g'(2) = 5\). So their speed at time \(t = 2\) is \(\sqrt{0^2 + 5^2} = 5\) miles per hour.

- **c.** [4 points] Write, but do not compute, an expression involving one or more integrals that gives the distance they traveled, in miles, in the first half hour of their trip.

  **Solution:** On the interval \((0, 0.5)\), we see that \(f(t) = 6t\), so on this interval, we have \(f'(t) = 6\) and \(g'(t) = -3t^2 + 10t - 3\).

  The parametric arc length formula then implies that the distance they travelled from \(t = 0\) to \(t = 0.5\) is \(\int_{0}^{0.5} \sqrt{(6)^2 + (-3t^2 + 10t - 3)^2} \, dt\) miles.

- **d.** [4 points] Write down a pair of parametric equations using the parameter \(s\) for the line tangent to their path at \(t = 2.75\) hours.

  **Solution:** Note that \(f(2.75) = -1.5, \frac{df}{dt}_{t=2.75} = -2, g(2.75) = 8.765625, \) and \(\frac{dg}{dt}_{t=2.75} = 1.8125\).

  There are many possible parametrizations. There is no need to have this match with the parameter \(t\) from earlier, so the answer below has the line passing through \((-1.5, 8.765625)\) at \(s = 0\).

  **Answer:** \(x(s) = -2s - 1.5\) and \(y(s) = 1.8125s + 8.765625\)