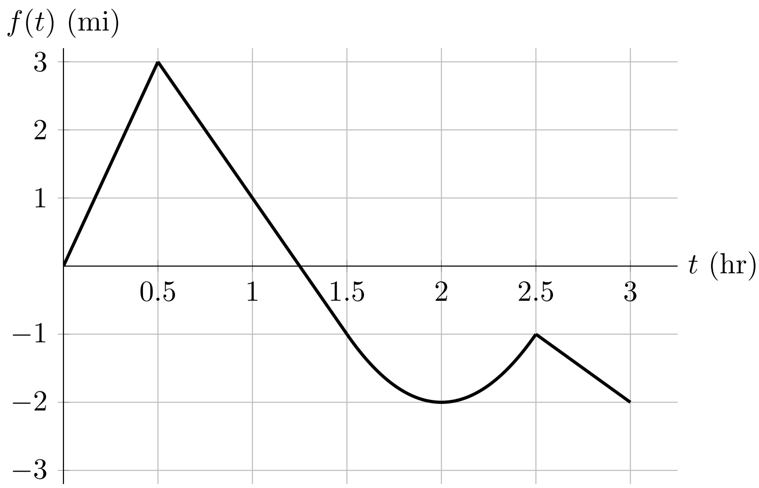
6. [13 points] Anderson and Glen decide to take a road trip starting from Venice Beach. They have no worries about getting anywhere quickly, as they enjoy each other's company, so they take a very inefficient route. Suppose that Venice Beach is located at $(0,0)$ and that Anderson and Glen's position (x, y) (measured in miles) t hours after leaving Venice Beach is given by a pair of parametric equations $x = f(t)$, $y = g(t)$. A graph of $f(t)$ and a formula for $g(t)$ are given below. Note that $f(t)$ is linear on the intervals $[0, 0.5]$, $[0.5, 1.5]$, and $[2.5, 3]$.



$$g(t) = -t^3 + 5t^2 - 3t$$

Note: For each of the following, your final answer should **not** involve the letters f and g.

a. [2 points] If their roadtrip last 3 hours, what are the $x-$ and $y-$ coordinates of their final destination?

Solution: Note that at time $t = 3$, we have $x = f(3) = -2$ and $y = g(3) = 9$. So the coordinates of their final destination are $(-2, 9)$.

b. [3 points] At what speed are they traveling 2 hours into their trip?

Solution: We have $\left.\frac{dx}{dt}\right|_{t=2} = f'(2) = 0$ and $\left.\frac{dy}{dt}\right|_{t=2} = g'(2) = 5$.
So their speed at time $t = 2$ is $\sqrt{0^2 + 5^2} = 5$ miles per hour.

c. [4 points] Write, but do not compute, an expression involving one or more integrals that gives the distance they traveled, in miles, in the first **half** hour of their trip.

Solution: On the interval $(0, 0.5)$, we see that $f(t) = 6t$, so on this interval, we have

$$f'(t) = 6 \quad \text{and} \quad g'(t) = -3t^2 + 10t - 3.$$

The parametric arc length formula then implies that the distance they travelled from $t = 0$ to $t = 0.5$ is $\int_0^{0.5} \sqrt{(6)^2 + (-3t^2 + 10t - 3)^2}\, dt$ miles.

d. [4 points] Write down a pair of parametric equations using the parameter s for the line tangent to their path at $t = 2.75$ hours.

Solution: Note that

$$f(2.75) = -1.5, \quad \left.\frac{df}{dt}\right|_{t=2.75} = -2, \quad g(2.75) = 8.765625, \quad \text{and} \quad \left.\frac{dg}{dt}\right|_{t=2.75} = 1.8125$$

There are many possible parametrizations. There is no need to have this match with the parameter t from earlier, so the answer below has the line passing through $(-1.5, 8.765625)$ at $s = 0$.

Answer: $x(s) =$ $-2s - 1.5$ and $y(s) =$ $1.8125s + 8.765625$