

7. [10 points] Consider the two sequences  $a_n$  and  $b_n$  defined by

$$a_n = \frac{1}{2^n} \qquad b_0 = 5, \quad b_n = 3b_{n-1} \text{ for all } n > 1.$$

Compute the following limits. If the sequence diverges, write DIVERGES.

**No justification necessary.**

a. [2 points]  $\lim_{n \rightarrow \infty} a_n$

**Answer:**  $\lim_{n \rightarrow \infty} a_n =$  \_\_\_\_\_

b. [2 points]  $\lim_{n \rightarrow \infty} \sum_{k=0}^n a_k$

**Answer:**  $\lim_{n \rightarrow \infty} \sum_{k=0}^n a_k =$  \_\_\_\_\_

c. [2 points]  $\lim_{n \rightarrow \infty} a_n b_n$

**Answer:**  $\lim_{n \rightarrow \infty} a_n b_n =$  \_\_\_\_\_

d. [2 points]  $\lim_{n \rightarrow \infty} \frac{\ln(b_n)}{\ln(a_n)}$

**Answer:**  $\lim_{n \rightarrow \infty} \frac{\ln(b_n)}{\ln(a_n)} =$  \_\_\_\_\_

e. [2 points]  $\lim_{n \rightarrow \infty} \frac{1 - e^{3a_n}}{a_n}$

**Answer:**  $\lim_{n \rightarrow \infty} \frac{1 - e^{3a_n}}{a_n} =$  \_\_\_\_\_