10. [10 points] Suppose $Q(x)$ is the cumulative distribution function (cdf) for a variable x, such that

$$Q(x) = \begin{cases} a & \text{for } x \leq 0 \\ b - e^{-cx} & \text{for } x > 0, \end{cases}$$

and the median value of x is 2.

a. [2 points] Let $q(x)$ be the probability density function for x. Write a formula for $q(x)$, assuming $q(0) = 0$.

Answer:

Solution: $Q(x)$ is an antiderivative of $q(x)$, so we can differentiate $Q(x)$ to get

$$q(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ ce^{-cx} & \text{for } x > 0 \end{cases}$$

b. [4 points] Set up, but <u>do not evaluate</u>, an expression involving one or more integrals that represents the mean value of x. Your answer may contain a, b, or c, but should <u>not</u> contain any function names (such at Q or q).

Answer: $\int_0^\infty cxe^{-cx}\,dx$

c. [4 points] Find the values of a, b, and c. Justify your answers, and write them in exact form. Remember that the <u>median</u> value of x is 2.

Solution: We know $\lim_{x\to-\infty} Q(x) = a = 0$ and $\lim_{x\to\infty} Q(x) = b = 1$. Finally, we have

$$\begin{aligned} Q(2) &= 1/2 \\ 1 - e^{2c} &= 1/2 \\ e^{-2c} &= 1/2 \\ -2c &= \ln(1/2) \\ c &= -\ln(1/2)/2 = \ln(2)/2 \end{aligned}$$

Answer: $a =$ **0** $b =$ **1** $c =$ $\ln(2)/2$