

3. [10 points] Consider the power series

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{5^n (2n)!} (x-9)^n.$$

- a. [1 point] What is the center of the interval of convergence of this power series?

Answer: $x = 9$

- b. [5 points] What is the radius of convergence of this power series? Show your work.

Solution:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left(\frac{((n+1)!)^2}{5^{n+1} (2n+2)!} \right) \left(\frac{5^n (2n)!}{(n!)^2} \right) = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{5(2n+2)(2n+1)} = \frac{1}{20}.$$

Therefore $R = 1/(\frac{1}{20}) = 20$.

Answer: Radius of convergence = 20

- c. [4 points] A certain power series $\sum_{n=1}^{\infty} C_n (x-4)^n$ converges when $x = 1$ and diverges when $x = 13$. Which of the following could be the radius of converge of this series? Circle all possibilities from the list below.

0 1 3 7 9 13 ∞ NONE OF THESE