

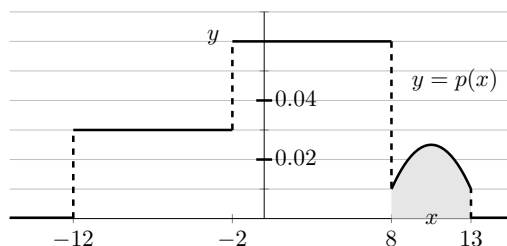
5. [12 points]

Yennifer’s Introductory Thermodynamics of Muck course is supposed to start at **9:10 am**, but her instructor does not consistently start on time. Let $p(x)$ be the probability density function for the amount of time x , in minutes, between when the instructor is supposed to start the class and when they actually start class.

Note that:

- $x = 0$ represents class starting at 9:10 am.
- A negative value of x represents starting class early.
- All of the nonzero portion of $p(x)$ is given in the graph below.
- The area of the shaded region is 0.1.

For parts **a.-c.**, you do not need to justify your answer.



a. [2 points] Yennifer is coming from another class and therefore always arrives at 9:06, exactly 4 minutes before class is supposed to start. Find the probability that class starts before Yennifer arrives.

Answer: 8 · 0.03 = 0.24 = 24%

b. [3 points] Which of the following statements is best supported by the equation $p(12) = 0.02$? Circle the **one** best answer.

- i. The probability that the instructor will start class at 9:22 is 2%.
- ii. The probability that the instructor will start class between 9:21 and 9:23 is about 2%.
- iii. The probability that the instructor will start class between 9:21 and 9:23 is about 4%.
- iv. The probability that the instructor has started class by 9:22 is about 2%.
- v. The probability that the instructor has started class by 9:22 is about 48%.

c. [3 points] Let $P(x)$ be the cumulative distribution function for $p(x)$. Which of the following could be the formula for $P(x)$ **on the interval** $-2 < x < 8$? Circle **all** answers that could be correct.

- | | | |
|----------------|--------------------------|------------------------------------------------------------------------------------------------------|
| i. $P(x) = 0$ | iii. $P(x) = 0.06x$ | v. $P(x) = 0.06(x + 2) + 0.3$ |
| ii. $P(x) = 1$ | iv. $P(x) = 0.06(x + 2)$ | vi. $P(x) = 0.1 - 0.06(x - 8)$ |

d. [4 points] Find the median value of x . Show your work, and write your answer in exact form.

Solution: We want to find x so that $\int_{-\infty}^x p(t) dt = 0.5$. The area under the curve for $x \leq -2$ is $0.03 \cdot 10 = 0.3$, so we want x with $-2 \leq x \leq 8$ such that $0.3 + .06(x + 2) = 0.5$. Solving for x , we find $x = 0.2/0.06 - 2 = \frac{10}{3} - 2 = \frac{4}{3}$.

Answer: 4/3