7. [10 points] Consider the two sequences a_n and b_n defined by

$$a_n = \frac{1}{2^n} \qquad\qquad b_0 = 5, \quad b_n = 3b_{n-1} \text{ for all } n > 1.$$

Compute the following limits. If the sequence diverges, write DIVERGES.
No justification necessary.

a. [2 points] $\lim_{n\to\infty} a_n$

Answer: $\lim_{n\to\infty} a_n =$ **0**

b. [2 points] $\lim_{n\to\infty} \sum_{k=0}^{n} a_k$

Answer: $\lim_{n\to\infty} \sum_{k=0}^{n} a_k =$ **2**

c. [2 points] $\lim_{n\to\infty} a_n b_n$

Answer: $\lim_{n\to\infty} a_n b_n =$ ∞ **or** DIVERGES

d. [2 points] $\lim_{n\to\infty} \frac{\ln(b_n)}{\ln(a_n)}$

Answer: $\lim_{n\to\infty} \frac{\ln(b_n)}{\ln(a_n)} =$ $\frac{\ln(3)}{\ln(1/2)} \approx -1.585$

e. [2 points] $\lim_{n\to\infty} \frac{1-e^{3a_n}}{a_n}$

Solution: Since this is a limit of the form 0/0, we solve this by applying L'Hôpital's Rule.

Answer: $\lim_{n\to\infty} \frac{1-e^{3a_n}}{a_n} =$ -3