

9. [11 points] Leight Vloss had trouble paying his rent so he started a cult. His followers believe that Leight receives holy messages from “The Great Consciousness” hiding in the internet. Each month Leight recruits  $D$  new followers and loses 20% of the followers he had in the previous month to disillusionment and other cults. That is, the number of followers Leight has after  $n$  months is described by the recursive formula

$$F_n = D + .8F_{n-1}$$

- a. [4 points] Supposing that Leight has 0 followers the moment he gets the idea to start a cult, which is to say that  $F_0 = 0$ , compute the number of followers he has in the first three months. Your answer may be in terms of  $D$ .

**Answer:**  $F_1 = \underline{\hspace{10em} D \hspace{10em}}$

$$F_2 = \underline{\hspace{10em} D + .8D = 1.8D \hspace{10em}}$$

$$F_3 = \underline{\hspace{10em} D + .8D + (.8)^2D = 2.44D \hspace{10em}}$$

- b. [4 points] Find a closed form expression for  $F_n$ , the number of followers Leight has after  $n$  months of channelling the spirit of the internet.

*Solution:* As we can see above,  $F_n$  is a finite geometric series with initial term  $D$  and ratio 0.8.

**Answer:**  $F_n = \underline{\hspace{10em} D \cdot \frac{1 - .8^n}{1 - .8} \hspace{10em}}$

- c. [3 points] Leight finds he needs the number of followers to tend to 1000 in the long run to ensure he can make rent each month. What’s the fewest number of followers  $D$  that Leight Vloss needs to recruit each month to make sure he can pay rent?

*Solution:* Leight needs

$$\lim_{n \rightarrow \infty} F_n \geq 1000.$$

So

$$\lim_{n \rightarrow \infty} F_n = \lim_{n \rightarrow \infty} D \frac{1 - .8^n}{1 - .8} = \frac{D}{.2} \implies D \geq 200.$$

**Answer:**  $\underline{\hspace{10em} 200 \hspace{10em}}$