10. [9 points] Let $f(x)$ be a continuous, decreasing, positive function defined for all $0 < x < \infty$ such that

- $\int_0^1 f(x)\,dx$ diverges and
- $\int_1^\infty f(x)\,dx$ converges.

Define $a_n = f(n)$ and $S_n = a_1 + a_2 + \cdots + a_n$ for $n \geq 1$.

Determine whether each of the following must converge, must diverge, or whether convergence cannot be determined. You do not need to justify your answers.

i. The sequence a_n **Converges** **Diverges** **Cannot be determined**

ii. The series $\sum_{n=1}^{\infty} a_n$ **Converges** **Diverges** **Cannot be determined**

iii. The sequence S_n **Converges** **Diverges** **Cannot be determined**

iv. The series $\sum_{n=1}^{\infty} S_n$ **Converges** **Diverges** **Cannot be determined**

v. $\int_0^\infty f(x)\,dx$ **Converges** **Diverges** **Cannot be determined**

vi. The series $\sum_{n=1}^{\infty} f(1/n)$ **Converges** **Diverges** **Cannot be determined**

vii. $\int_0^1 \frac{f(1/x)}{x^2}\,dx$ **Converges** **Diverges** **Cannot be determined**