

4. [9 points] Michigan Atomic and Thermonuclear Headquarter (M.A.T.H.) recently discovered a new chemical element X, which is radioactive with a half-life of 1 day. Currently, the M.A.T.H. lab is scheduled to synthesize  $k$  grams of X everyday at noon. Let  $m_n$  be the mass (in grams) of X the M.A.T.H. lab has in possession at noon on the  $n$ th day of production, *immediately after* the new batch is produced; for example,  $m_1 = k$ .

a. [2 points] Calculate  $m_2$  and  $m_3$ .

**Answer:**  $m_2 =$  \_\_\_\_\_

**Answer:**  $m_3 =$  \_\_\_\_\_

b. [4 points] Find a closed form expression for  $m_n$ .

**Answer:**  $m_n =$  \_\_\_\_\_

- c. [3 points] The M.A.T.H lab plans to conduct an experiment on the element X which requires having 10 grams of X at once. At this production level, for what values of  $k$  can the experiment be carried out at some point in the future?

**Answer:** \_\_\_\_\_