7. [10 points]

a. [5 points] Determine the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n^2(2n)!}{2^n(n!)^2} x^{2n}$.

Answer: _____

b. [5 points] You do not need to justify your answers below.
Suppose C_n is a sequence such that the following are true:

- C_n is a monotone decreasing sequence
- C_n converges to 0
- $\displaystyle\sum_{n=0}^{\infty} C_n$ diverges
- $\displaystyle\sum_{n=0}^{\infty} \frac{C_n(x+3)^n}{4^n}$ has radius of convergence 4.

What is the center of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{C_n(x+3)^n}{4^n}$?

Answer: _____

What are the endpoints of the interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{C_n(x+3)^n}{4^n}$?

Answer: Left endpoint at $a =$ _____

Right endpoint at $b =$ _____

Let a and b be the left and right endpoints of the interval of convergence you found above. Which of the following could be the interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{C_n(x+3)^n}{4^n}$?

$[a,b]$ $\qquad\qquad$ $(a,b]$ $\qquad\qquad$ (a,b) $\qquad\qquad$ $[a,b)$