

1. [12 points] The parts of this problem are unrelated. You do not need to justify your answers.
- a. [6 points] For each of the following sequences, defined for $n \geq 1$, decide if it is bounded, if it is increasing or decreasing, and whether it converges, and circle your answers. If it converges, find the limit.

i. $b_n = \frac{2n + e^{-n}}{5n}$

Bounded

Increasing

Decreasing

Diverges

Converges to

2/5

ii. $c_n = \sin(n)$

Bounded

Increasing

Decreasing

Diverges

Converges to _____

iii. $d_n = \sum_{k=1}^n \frac{3}{k}$

Bounded

Increasing

Decreasing

Diverges

Converges to _____

- b. [3 points] Write the following series using sigma notation:

$$2^3(x - e)^4 + 3^3(x - e)^6 + 4^3(x - e)^8 + \dots$$

Answer:

$$\sum_{n=2}^{\infty} n^3(x - e)^{2n} = \sum_{n=1}^{\infty} (n+1)^3(x - e)^{2(n+1)}$$

- c. [3 points] Suppose the power series $\sum_{n=0}^{\infty} C_n(x - 2)^n$ converges at $x = 5$ and diverges at $x = 9$. Which of the following could be the radius of convergence R ? Circle all correct answers.

$R = 0$

$R = 2$

$R = 3$

$R = 4$

$R = 7$

$R = 8$

$R = 10$