- 10. [9 points] Let f(x) be a continuous, decreasing, positive function defined for all $0 < x < \infty$ such that
 - $\int_0^1 f(x) dx$ diverges and
 - $\int_{1}^{\infty} f(x) dx$ converges.

Define $a_n = f(n)$ and $S_n = a_1 + a_2 + \cdots + a_n$ for $n \ge 1$.

Determine whether each of the following must converge, must diverge, or whether convergence cannot be determined. You do not need to justify your answers.

i. The sequence a_n

Converges

Diverges

Cannot be determined

ii. The series $\sum_{n=1}^{\infty} a_n$

Converges

Diverges

Cannot be determined

iii. The sequence S_n

Converges

Diverges

Cannot be determined

iv. The series $\sum_{n=1}^{\infty} S_n$

Converges

Diverges

Cannot be determined

v. $\int_0^\infty f(x) dx$

Converges

Diverges

Cannot be determined

vi. The series $\sum_{n=1}^{\infty} f(1/n)$

Converges

Diverges

Cannot be determined

vii. $\int_0^1 \frac{f(1/x)}{x^2} dx$

Converges

Diverges

Cannot be determined