**4**. [9 points] Michigan Atomic and Thermonuclear Headquarter (M.A.T.H.) recently discovered a new chemical element X, which is radioactive with a half-life of 1 day. Currently, the M.A.T.H. lab is scheduled to synthesize $k$ grams of X everyday at noon.
Let $m_n$ be the mass (in grams) of X the M.A.T.H. lab has in possession at noon on the $n$th day of production, *immediately after* the new batch is produced; for example, $m_1 = k$.

**a**. [2 points] Calculate $m_2$ and $m_3$.

**Answer:** $m_2 =$ $\frac{k}{2} + k$

**Answer:** $m_3 =$ $\frac{k}{4} + \frac{k}{2} + k$

**b**. [4 points] Find a closed form expression for $m_n$.

**Answer:** $m_n =$ $\frac{k\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}} = 2k\left(1 - \left(\frac{1}{2^n}\right)\right)$

**c**. [3 points] The M.A.T.H lab plans to conduct an experiment on the element X which requires having 10 grams of X at once. At this production level, for what values of $k$ can the experiment be carried out at some point in the future?

*Solution:* Since

$$\lim_{n\to\infty} m_n = 2k,$$

and we need $m_n \geq 10$ for some $n$, it follows that $2k > 10$ and therefore $k > 5$.

**Answer:** $k > 5$