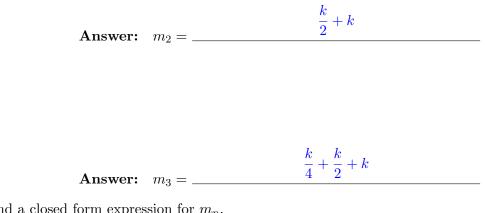
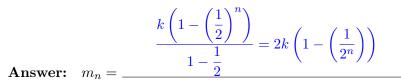
4. [9 points] Michigan Atomic and Thermonuclear Headquarter (M.A.T.H.) recently discovered a new chemical element X, which is radioactive with a half-life of 1 day. Currently, the M.A.T.H. lab is scheduled to synthesize k grams of X everyday at noon.

Let m_n be the mass (in grams) of X the M.A.T.H. lab has in possession at noon on the *n*th day of production, *immediately after* the new batch is produced; for example, $m_1 = k$.

a. [2 points] Calculate m_2 and m_3 .



b. [4 points] Find a closed form expression for m_n .



c. [3 points] The M.A.T.H lab plans to conduct an experiment on the element X which requires having 10 grams of X at once. At this production level, for what values of k can the experiment be carried out at some point in the future?

Solution: Since

$$\lim_{n \to \infty} m_n = 2k,$$

and we need $m_n \ge 10$ for some n, it follows that 2k > 10 and therefore k > 5.

Answer:

k > 5