6. [9 points] For each of the following questions, circle all answers that must be correct.

a. [3 points] Circle all true statements. The integral $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}+x^2}\,dx$

i. diverges because $\dfrac{1}{\sqrt{x}+x^2} > \dfrac{1}{2\sqrt{x}}$ for $0 < x < 1$.

ii. diverges because $\dfrac{1}{\sqrt{x}+x^2} > \dfrac{1}{2x^2}$ for $1 < x < \infty$.

iii. converges because $\displaystyle\lim_{x\to\infty} \frac{1}{\sqrt{x}+x^2} = 0$.

iv. converges by p-test with $p = 2$.

v. None of these.

b. [3 points] Consider a geometric series with n^{th} partial sum S_n, where $\displaystyle\lim_{n\to\infty} S_n = \frac{5}{1-0.3}$. Which of the following statements must be true?

i. This geometric series must converge.

ii. The first term of this geometric series must be 0.3.

iii. The common ratio of this geometric series must be 0.3.

iv. This geometric series may or may not converge; it cannot be determined.

v. None of these.

c. [3 points] The series $1 - \dfrac{1}{3} + \dfrac{1}{2} - \dfrac{1}{3^2} + \dfrac{1}{2^2} - \dfrac{1}{3^3} + \dfrac{1}{2^3} - \ldots$

i. converges by the Alternating Series Test.

ii. diverges because the Alternating Series Test does not apply.

iii. neither converges nor diverges.

iv. converges because it is a geometric series with common ratio of magnitude less than 1.

v. converges because the terms converge to 0.

vi. None of these.