9. [9 points] The blueprint for the Infinity Tower has been finalized, and the design of the Tower of Hanoi is accepted. Specifically:

- the tower will have infinitely many floors
- each floor has the shape of a solid cylinder of height of 3 meters
- the $n$th floor has radius $\frac{1}{2n^2}$ meters
- the ground floor corresponds to $n = 1$
- the tower has constant density $\delta$ kg/m$^3$
- when construction begins, all materials are on the ground and have to be lifted to build each floor.

In this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s$^2$.

a. [7 points] Let $W_n$ be the work, in Joules, it takes to lift the materials to build the $n$th floor and put that floor in place in the tower. Write an expression involving one or more integrals for each of the following.

i. $W_1 = \int_0^3 \pi \left( \frac{1}{2} \right)^2 \delta gh \, dh$

ii. $W_2 = \int_3^6 \pi \left( \frac{1}{8} \right)^2 \delta gh \, dh = \int_0^3 \pi \left( \frac{1}{8} \right)^2 \delta g(3 + h) \, dh$

iii. $W_n = \int_{3(n-1)}^{3n} \pi \left( \frac{1}{2n^2} \right)^2 \delta gh \, dh = \int_0^3 \pi \left( \frac{1}{2n^2} \right)^2 \delta g(3(n - 1) + h) \, dh$

b. [2 points] Write an expression involving one or more integrals and/or series that gives the total work it would take to build the entire tower. Your answer should not include the letter $W$.

Answer: $\sum_{n=1}^{\infty} \int_{3(n-1)}^{3n} \pi \left( \frac{1}{2n^2} \right)^2 \delta gh \, dh = \sum_{n=1}^{\infty} \int_0^3 \pi \left( \frac{1}{2n^2} \right)^2 \delta g(3(n - 1) + h) \, dh$