9. [9 points] The blueprint for the Infinity Tower has been finalized, and the design of the Tower of Hanoi is accepted. Specifically:

- the tower will have infinitely many floors
- each floor has the shape of a solid cylinder of height of 3 meters
- the nth floor has radius $\dfrac{1}{2n^2}$ meters
- the ground floor corresponds to $n = 1$
- the tower has constant density δ kg/m^3
- when construction begins, all materials are on the ground and have to be lifted to build each floor.

In this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2.

a. [7 points] Let W_n be the work, in Joules, it takes to lift the materials to build the nth floor and put that floor in place in the tower. Write an expression involving one or more integrals for each of the following.

i. $W_1 =$ $\displaystyle\int_0^3 \pi \left(\frac{1}{2}\right)^2 \delta g h \, dh$

ii. $W_2 =$ $\displaystyle\int_3^6 \pi \left(\frac{1}{8}\right)^2 \delta g h \, dh = \int_0^3 \pi \left(\frac{1}{8}\right)^2 \delta g (3+h) \, dh$

iii. $W_n =$ $\displaystyle\int_{3(n-1)}^{3n} \pi \left(\frac{1}{2n^2}\right)^2 \delta g h \, dh = \int_0^3 \pi \left(\frac{1}{2n^2}\right)^2 \delta g (3(n-1)+h) \, dh$

b. [2 points] Write an expression involving one or more integrals and/or series that gives the total work it would take to build the entire tower. Your answer should not include the letter W.

Answer: $\displaystyle\sum_{n=1}^{\infty} \int_{3(n-1)}^{3n} \pi \left(\frac{1}{2n^2}\right)^2 \delta g h \, dh = \sum_{n=1}^{\infty} \int_0^3 \pi \left(\frac{1}{2n^2}\right)^2 \delta g (3(n-1)+h) \, dh$