

9. [17 points]

- a. [4 points] Ivan is studying the series $\sum_{n=1}^{\infty} (-1)^n (1 + e^{-n})$ and writes the following argument:

The series is alternating. If we let $a_n = |(-1)^n (1 + e^{-n})| = 1 + e^{-n}$, then a_n is positive, decreasing, but $\lim_{n \rightarrow \infty} a_n = 1$ is not 0. Therefore $\sum_{n=1}^{\infty} (-1)^n (1 + e^{-n})$ diverges by the alternating series test.

Ivan's instructor tells Ivan that even though the $\sum_{n=1}^{\infty} (-1)^n (1 + e^{-n})$ does diverge, the above argument is incorrect. Explain what's wrong with this argument, and give a correct argument to show that $\sum_{n=1}^{\infty} (-1)^n (1 + e^{-n})$ diverges.

- b. [6 points] Determine whether the following series converge or diverge. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests or theorems you use.

$$\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$$

- c. [7 points] Determine whether the following series **converges absolutely, converges conditionally, or diverges**. Be sure to fully justify your answer, using **proper notation** and showing mechanics of any tests or theorems you use.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$$