10. [12 points] Show that the following statements are false by giving a concrete example to contradict each of the statement. You can write a formula or draw a clear, well-labeled graph in place of the blanks. Accompany your example with a brief but complete explanation.

a. [4 points] If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=1}^{\infty} a_n \) converges.

Give your answer in the form:

“The statement is false when \( a_n = \ldots \) because...”

Solution: For example, \( \lim_{n \to \infty} \frac{1}{n} = 0 \), but \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges by p-test, \( p = 1 \).

b. [4 points] For any continuous function \( f(x) \) with \( f(x) > 0 \), the improper integral \( \int_{-100}^{\infty} f(x) \, dx \) always diverges.

Give your answer in the form:

“The statement is false when \( f(x) = \ldots \) because...”

Solution: An example is \( f(x) = e^{-x} \), as \( \int_{-100}^{\infty} e^{-x} \, dx = e^{100} \). We can also see that the integral converges by exponential decay test.

c. [4 points] If \( P(x) \) is a cumulative distribution function, then \( P(0) = 0 \).

Give your answer in the form:

“The statement is false when \( P(x) = \ldots \) because...”

(Note: Your \( P(x) \) needs to be a cumulative distribution function, but you do not need to show/prove that it is.)
Solution: An example of $P(x)$ is given by the following graph.

In particular, $P(x)$ is indeed a cumulative distribution function, as $P(x)$ is increasing from 0 to 1 and it is continuous. However, $P(0) = 1 \neq 0$. 