10. [12 points] Show that the following statements are false by giving a concrete example to contradict each of the statement. You can write a formula or draw a clear, well-labeled graph in place of the blanks. Accompany your example with a brief but complete explanation.

a. [4 points] If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

Give your answer in the form:
"The statement is false when $a_n =$ ____________ because..."

Solution: For example, $\lim_{n\to\infty} \frac{1}{n} = 0$, but $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-test, $p = 1$.

b. [4 points] For any continuous function $f(x)$ with $f(x) > 0$, the improper integral $\int_{-100}^{\infty} f(x)\, dx$ always diverges.

Give your answer in the form:

"The statement is false when $f(x) =$ ____________ because..."

Solution: An example is $f(x) = e^{-x}$, as $\int_{-100}^{\infty} e^{-x}\, dx = e^{100}$. We can also see that the integral converges by exponential decay test.

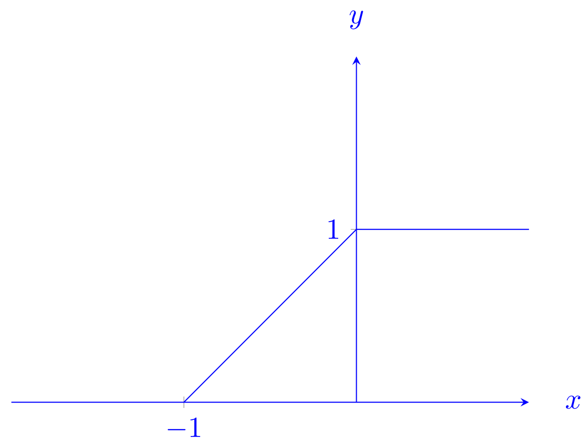
c. [4 points] If $P(x)$ is a cumulative distribution function, then $P(0) = 0$.

Give your answer in the form:

"The statement is false when $P(x) =$ ____________ because..."

(Note: Your $P(x)$ needs to be a cumulative distribution function, but you do not need to show/prove that it is.)

Solution: An example of $P(x)$ is given by the following graph.



In particular, $P(x)$ is indeed a cumulative distribution function, as $P(x)$ is increasing from 0 to 1 and it is continuous. However, $P(0) = 1 \neq 0$.