2. [12 points] In order to build a settlement on the island, intruders start cutting down trees at the forest, cutting the trees into logs, and putting the logs in a pile. Let A_n be the number of logs they have in the pile at noon on the n-th day. The intruders have 100 logs in the pile at noon on the first day (so $A_1 = 100$). Every day (between noon on one day and noon on the next day), the building team uses 10% of the logs in the pile, while the log-cutting team adds 20 logs to the pile immediately before noon.

a. [4 points] Find A_2 and A_3. **You do not need to simplify your answers.**

Solution:

$$A_2 = 100 \cdot 0.9 + 20$$

$$A_3 = (100 \cdot 0.9 + 20) \cdot 0.9 + 20 = 100 \cdot 0.9^2 + 20 + 20 \cdot 0.9$$

b. [5 points] Find a **closed form** expression for A_n. Closed form means your answer should not include ellipses or sigma notation, and should NOT be recursive. **You do not need to simplify your closed form answer.**

Solution: From observing the pattern from A_1, A_2 and A_3, we have

$$\begin{aligned} A_n &= 100 \cdot 0.9^{n-1} + (20 + 20 \cdot 0.9 + 20 \cdot 0.9^2 + \cdots + 20 \cdot 0.9^{n-2}) \\ &= 100 \cdot 0.9^{n-1} + 20 \cdot \frac{1 - 0.9^{n-1}}{1 - 0.9}. \end{aligned}$$

Note that the term $100 \cdot 0.9^{n-1}$ is not part of the geometric series. There are $n-1$ terms in the geometric series, so the exponent in the closed form is $n-1$.

c. [3 points] How many logs will the intruders have in the pile in the long run?

Solution:

$$\lim_{n\to\infty} 100 \cdot 0.9^{n-1} + 20 \cdot \frac{1 - 0.9^{n-1}}{1 - 0.9} = 0 + 20 \cdot \frac{1}{1 - 0.9} = 20 \cdot 10 = 200.$$