3. [7 points] Emily is a physics teacher. She is demonstrating the physics of rocket-launching with a water rocket in her class today. The water rocket weighs 3 lbs on the ground, and Emily launches it straight up to 10 ft above the ground. During the launch, the rocket's weight decreases at constant rate (in lbs/ft) as the water is ejected from the rocket. When it reaches 10 ft above the ground, the rocket weighs 1 lb.

a. [3 points] Calculate the weight of the rocket when it is a height h ft above the ground. Include units.

Solution: Since the rocket's weight decreases at a constant rate in lb / ft, the rocket's weight is a linear function of its height above the ground, i.e. h.
At $h = 0$, the weight is 3lb. At $h = 10$, the weight is 1lb. Hence the slope of weight in terms of height is

$$\frac{1-3}{10-0} = \frac{-1}{5}.$$

By using point-slope form with $h = 0$ and weight $= 3$lb, we have that

$$(\text{Weight at height } h) - 3 = \frac{-1}{5}(h-0),$$

$$\text{Weight at height } h = 3 - \frac{h}{5} \text{ lb}.$$

b. [4 points] Write an expression involving integrals for the total work required to propel the rocket from the ground to a height of 10 feet above the ground (as described above). Do not evaluate any integrals in your expression. Include units.

Solution: Work to lift the rocket from h ft above ground to $h + \Delta h$ ft above ground is

$$(\text{Weight at height } h)(\text{lb}) \cdot (\text{distance travelling})(\text{ft}) = (3 - \frac{h}{5})\Delta h(\text{ftlb}).$$

Note that there is no $g = 9.8$, since lb is a unit for force. Hence total work is

$$\int_0^{10} (3 - \frac{h}{5})\, dh \text{ ftlb}.$$