

4. [7 points] Some of Flora's friends are hurt while the intruders are building the settlement. Flora and Nile are trying their best to heal them. Suppose  $p(x)$  is the probability density function for the number of weeks,  $x$ , it takes for everyone to recover after intruders appear.

$$p(x) = \begin{cases} c & \text{if } 0 < x \leq 1, \\ 2c & \text{if } 1 < x \leq 3, \\ 0 & \text{else.} \end{cases}$$

- a. [2 points] Find  $c$ .

*Solution:*

$$\text{Area under full graph} = c \cdot (1 - 0) + 2c \cdot (3 - 1) = 5c.$$

Since  $p(x)$  is a probability density function, area under full graph = 1, so  $c = 1/5$ .

- b. [5 points] Let  $W(x)$  be the cumulative distribution function for  $p(x)$ . Showing your work, give a piece-wise defined formula for  $W(x)$  in the form given below.

$$W(x) = \begin{cases} \underline{\hspace{2cm}} & \text{if } x \leq 0, \\ \underline{\hspace{2cm}} & \text{if } 0 < x \leq 1, \\ \underline{\hspace{2cm}} & \text{if } 1 < x \leq 3, \\ \underline{\hspace{2cm}} & \text{if } x > 3. \end{cases}$$

*Solution:*

$$W(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ cx = \frac{x}{5} & \text{if } 0 < x \leq 1, \\ c + 2c(x - 1) = -\frac{1}{5} + \frac{2}{5}x & \text{if } 1 < x \leq 3, \\ 1 & \text{if } x > 3. \end{cases}$$

Explanation:

There is no area under the pdf before  $x = 0$ , so  $W(x) = 0$  if  $x \leq 0$ .

All area under the pdf is picked up after  $x = 3$ , so  $W(x) = 1$  if  $x > 3$ .

For  $0 < x \leq 1$ ,

$$W(x) = \int_0^x p(t) dt = cx.$$

For  $1 < x \leq 3$ ,

$$W(x) = \int_0^x p(t) dt = \int_0^1 p(t) dt + \int_1^x p(t) dt = c + 2c(x - 1).$$