7. [6 points] Determine whether the following improper integral converges or diverges. **Fully justify** your answer including using **proper notation**, and showing mechanics of any tests or theorems you use.

$$\int_0^1 \frac{\pi}{x^3 + \sqrt{x}}\,dx$$

Solution: This integral is improper at the lower bound 0, since the denominator is 0 at $x = 0$. Consider the dominating terms in the denominator. As $x \to 0^+$, $\sqrt{x}$ dominates x^3. As a result we should compare the integrand to $\pi/\sqrt{x}$.
In any case, since both x^3 and $\sqrt{x}$ are positive, if we take x^3 away, the denominator gets smaller. Hence the fraction gets bigger. Thus,

$$0 < \frac{\pi}{x^3 + \sqrt{x}} \le \frac{\pi}{\sqrt{x}} \qquad \text{for } 0 < x \le 1.$$

The integral

$$\int_0^1 \frac{\pi}{\sqrt{x}}\,dx$$

converges by p-test, $p = 1/2$. Therefore, by comparison test,

$$\int_0^1 \frac{\pi}{x^3 + \sqrt{x}}\,dx$$

converges.