

2. [7 points] Brad and Joan are examining another pitcher's probability density function (pdf) when Brad spills coffee on the paper and smudges some of the ink. After drying off the paper, they are left with the incomplete probability density function, $g(v)$ given below, where v is in **hundreds of miles per hour**.

$$g(v) = \begin{cases} r + qv & 0 < v \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Brad and Joan know that this player has a mean pitch speed of $\frac{2}{3}$ hundreds of miles per hour. Find the values of r and q which make this function a probability density function.

Solution: We need to find r and q such that the above function is a probability density function with the given mean. This means we need the equations

$$\int_0^1 r + qv dv = 1 \qquad \int_0^1 v(r + qv) dv = \frac{2}{3}$$

to be true. If we compute each integral, we get the equations

$$\int_0^1 r + qv dv = rv + \frac{qv^2}{2} \Big|_0^1 = r + \frac{q}{2}$$

and

$$\int_0^1 v(r + qv) dv = \frac{rv^2}{2} + \frac{qv^3}{3} \Big|_0^1 = \frac{r}{2} + \frac{q}{3}$$

so we just solve

$$r + \frac{q}{2} = 1 \qquad \frac{r}{2} + \frac{q}{3} = \frac{2}{3}$$

Solving this gives $r = 0$, $q = 2$

$$r = \underline{\quad 0 \quad}$$

$$q = \underline{\quad 2 \quad}$$