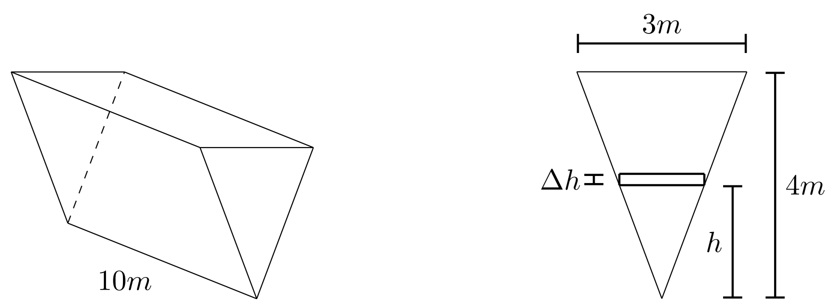
4. [15 points] A gas station needs to pump gas out of a subterranean tank. The tank is 10 meters in length, and has cross-sections shaped like isosceles triangles, with base 3 meters and height 4 meters. **The top of the tank is 15 meters below the surface of the earth**. Recall that $g = 9.8m/s^2$ is the gravitational constant.



Underground Tank

a. [5 points] Write an expression for the volume (in cubic meters) of a horizontal rectangular slice of gasoline at height h above the bottom of the tank, with thickness Δh. Your answer should not involve an integral.

Solution: The slice has volume $\ell w \Delta h$. The length is constant at $10m$, so we just need to find the width as a function of height. Using similar triangles, $\frac{w}{h} = \frac{3}{4}$ so $w = \frac{3}{4}h$. Therefore the slice volume is $\frac{15}{2}h\Delta h\ m^3$.

b. [3 points] Gasoline has a density of 800 kg/m^3. Write an expression for the weight (in newtons) of the slice of gasoline mentioned in part (a). Your answer should not involve an integral.

Solution: Weight is the force exerted on a mass due to gravity, and so weight is mg. We compute the mass using the density from the problem statement and the volume from a). This means $m = (800)(\frac{15}{2}h)\Delta h kg$. Then the weight is

$$(9.8)(800)\left(\frac{15}{2}h\right)\Delta h\ N$$

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c. [4 points] Write an expression for the work (in joules) needed to pump the slice of gasoline mentioned above to the surface of the earth. Your answer should not involve an integral.

Solution: Work is force times distance traveled. If the slice is h meters from the bottom of the tank, then the slice travels $(4 - h)$ meters to get to the top of the tank. Then it travels the 15 meters to get from the top of the tank to the ground. Therefore, the total distance traveled is $19 - h$ meters. The force is the weight from b), so the work for a slice is:

$$W_{\text{slice}} = (19 - h)(9.8)(800)\left(\frac{15}{2}h\right)\Delta h$$

d. [3 points] Write an integral for the total work (in joules) needed pump all of the gasoline to the surface of the earth.

Solution: We integrate our work slices to find the total work. Since the slices range from $h = 0$ to $h = 4$, these are the bounds, so the work is given by the integral

$$\int_0^4 (19 - h)(9.8)(800)\left(\frac{15}{2}h\right)\ dh$$