5. [12 points]   A tech startup is growing quickly, and the company needs to understand its customers data-storage needs to properly scale its infrastructure. Over the course of each month, the users each store 5 gigabytes of new data. Additionally, because users are conscious of their digital footprint, at the beginning of each month, each user deletes 20% of all data they had stored in previous months.

a. [4 points] Let D_n be the amount of data stored per user at the end of the nth month. If $D_1 = 5$, write expressions for D_2 and D_3. The letter D should not appear in your final answers.

$$D_2 = \underline{\quad \dfrac{5 + 5\,(.8)}{} \quad}$$

$$D_3 = \underline{\quad \dfrac{5 + 5\,(.8) + 5\,(.8)^2}{} \quad}$$

b. [4 points] Find a closed form expression for D_n. This means your answer should be a function of n, should not contain Σ, and should not be recursive.

$$D_n = \underline{\quad \dfrac{\frac{5(1 - (.8)^n)}{1 - (.8)}}{} \quad}$$

c. [4 points] What is the long-term expected data storage of a user in gigabytes?

$$\text{Answer} = \underline{\quad \dfrac{\frac{5}{1 - .8} = 25}{} \quad}$$