7. [12 points] The parts of this problem are unrelated to each other.

a. [7 points] **Compute** the value of the following improper integral if it converges. If it does not converge, use a **direct computation** of the integral to show its divergence. Be sure to show your full computation, and be sure to use **proper notation.**

$$\int_1^2 \frac{1}{\sqrt{t-1}}dt$$

Solution: First, this is an improper integral at $t = 1$. Therefore, we need to switch to limit notation:

$$\int_1^2 \frac{1}{\sqrt{t-1}}dt = \lim_{b\to 1^+} \int_b^2 \frac{1}{\sqrt{t-1}}dt$$

Now, we do a u-sub, with $u = t - 1$, so $du = dt$, so our integral becomes

$$\lim_{b\to 1^+} \int_{b-1}^1 \frac{1}{\sqrt{u}}du = \lim_{b\to 1^+} 2\sqrt{u}\Big|_{b-1}^1$$

Evaluating we get:

$$\lim_{b\to 1^+} (2\sqrt{1} - 2\sqrt{b-1}) = 2$$

b. [5 points] Compute the following limit. Fully justify your answer including using proper notation.

$$\lim_{x\to 0} \quad \frac{1-\cos(x)}{x^2}$$

Solution: As x goes to zero, this becomes an indeterminant form of $\frac{0}{0}$, so we apply L'Hopital to get

$$\lim_{x\to 0} \quad \frac{1-\cos(x)}{x^2} = \lim_{x\to 0} \frac{\sin(x)}{2x}.$$

This is also an indeterminant form, so we use L'Hopital again, to get:

$$\lim_{x\to 0} \frac{\sin(x)}{2x} = \lim_{x\to 0} \frac{\cos(x)}{2}.$$

Computing the final limit gives

$$\lim_{x\to 0} \frac{\cos(x)}{2} = \frac{\cos(0)}{2} = \frac{1}{2}.$$

So the final answer is

$$\lim_{x\to 0} \quad \frac{1-\cos(x)}{x^2} = \frac{1}{2}$$