8. [8 points] Determine whether the following improper integral converges or diverges. **Circle your final answer choice**. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\int_1^\infty \frac{t^2+\ln(t)}{t^3-\cos(t)+2}dt$$

Circle one: **Converges** **Diverges**

Solution: The numerator of the integrand is dominated by t^2, and the denominator is dominated by t^3, so this function has the same behavior as $\frac{t^2}{t^3}=\frac{1}{t}$, so we expect it to diverge. Therefore, we want to bound this function below by a function whose integral diverges. First, we note that $t^2 \le t^2+\ln(t)$ on $[1,\infty)$. Then, for the denominator, since $\cos(x)$ oscillates from $[-1,1]$, the denominator is largest(ans so the function is smallest) when $\cos(x)=-1$, so we get that $t^3-\cos(t)+2 \le t^3+1+2$, and so

$$\frac{t^2}{t^3+3} \le \frac{t^2}{t^3-\cos(t)+2} \le \frac{t^2+\ln(t)}{t^3-\cos(t)+2}$$

Next we know that $3 \le \frac{1}{2}t^3$ on $[2,\infty]$, and so $t^3+3 \le t^3\frac{1}{2}t^3 = \left(\frac{3}{2}\right)t^3$, and so we get

$$\left(\frac{2}{3}\right)\frac{1}{t} = \left(\frac{2}{3}\right)\frac{t^2}{t^3} \le \frac{t^2}{t^3+3} \le \frac{t^2+\ln(t)}{t^3-\cos(t)+2}.$$

Then, $\frac{2}{3}\int_1^\infty \frac{1}{t}dt$ diverges by p-test, with $p=1$, and so $\int_1^\infty \frac{t^2+\ln(t)}{t^3-\cos(t)+2}dt$ diverges by comparison test, comparing $\left(\frac{2}{3}\right)\frac{1}{t} \le \frac{t^2+\ln(t)}{t^3-\cos(t)+2}$